STATISTICAL APPROACH FOR ESTIMATING INTERVALS OF CERTIFICATION OR BIASES OF FACILITIES OR MEASUREMENT SYSTEMS INCLUDING UNCERTAINTIES

by

F. Stern*, A. Olivieri[+], J. Shao*, J. Longo*, and T. Ratcliffe[#]

*****_IIHR—Hydroscience & Engineering, The University of Iowa, Iowa City, IA, 52242, USA_

[+]_Italian Ship Model Basin, Rome, Italy_

[#]_David Taylor Model Basin, Bethesda, MD_

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ABSTRACT

A statistical approach for estimating intervals of certification or biases of facilities or measurement systems including uncertainties is set forth based on MxN-order level testing, which is defined as M repetitions of the same N-order level experiment in M different facilities or in the same facility with M different measurement systems. In absence of reference values, mean facility or measurement system used for assessing intervals of certification or biases. Certification or biases of facilities or measurement systems are defined as processes for assessing probabilistic confidence intervals for facilities or measurement systems for specific tests, data reduction equations, conditions, procedures, and uncertainty analysis. Similarly, subgroup analysis performed for isolating and assessing levels of differences due to use of different model sizes (scale effects) or measurement systems. An example provided for towing tank facilities for resistance tests using standard uncertainty analysis procedures based on an international collaboration between three facilities. Although number of facilities minimum, the results demonstrate usefulness of approach and support recommendation of future collaborations between more facilities. Knowledge of intervals of certification or biases is important for design, accrediting facilities or measurement systems, and CFD validation.

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STATISTICAL APPROACH FOR ESTIMATING INTERVALS OF CERTIFICATION OR BIASES OF FACILITIES OR MEASUREMENT SYSTEMS INCLUDING UNCERTAINTIES

1. INTRODUCTION

Experimental fluid dynamics (EFD) testing in large-scale facilities at research institutes is undergoing change from routine tests for global variables to detailed tests for local variables for model development and computational fluid dynamics (CFD) validation, as design methodology changes from model testing and theory to simulation-based design. Detailed testing requires facilities utilize advanced modern measurement systems (MS) with complete documentation of test conditions, procedures, and uncertainty analysis. The requirements for intervals of uncertainties are even more stringent than required previously since they are a limiting factor in establishing intervals of CFD simulation validation [1] and code certification [2] and ultimately credibility of simulation technology. In addition, routine test data more likely utilized in house, whereas detailed test data is more likely utilized internationally, which additionally requires use of standard procedures and uncertainty analysis and establishment of benchmark intervals of uncertainties. Detailed testing offers new opportunities for research institutes, as the amount and complexity of testing is increased.

Methodology and procedures for estimating EFD uncertainties have developed and progressed over the past fifty years. Formalization [3-5] followed by standard procedures with emphasis on simplification and practical application [6-7]. However, rigorous use continues to be a problem in both research and design at university, industry, and government laboratories. Another problem is lack of methodology and procedures for estimating intervals of certification or biases of facilities or MS, i.e., establishing intervals of confidence for facilities or MS arising from systematic errors due to differences or peculiarities in individual facilities or MS. Such differences or peculiarities arise from detailed facility geometry or MS design, working fluid and flow quality, conditions and procedures, test engineers, specific locations, etc. Adding to the problem is fact that such developments require considerable resources and, in case of facilities, cooperation amongst institutes, which often crosses international boundaries. Estimating intervals of certification or biases of facilities or measurement systems are required for establishing standard intervals of uncertainties for various types of facilities (towing tanks, wind tunnels, flumes, etc.) and tests (forces and moments, motions, waves elevations, mean velocities, turbulence, etc.) and MS (load cells, potentiometers, wave probes, pitot, LDV, PIV, etc.). This is important for design, accrediting facilities or measurement systems, and CFD validation.

Most work on facility or MS biases is for small-scale flow meter calibration facilities with focus on validation of accuracy, comparison of international flow standards, and establishing domestic flow traceability [8]. Proficiency testing programs are used to establish flow measurement traceability, which are largely based on Youden plots [9] requiring two (e.g., tandem and/or upstream and downstream) MS at each facility. This approach not easily extended to large-scale multi-purpose facilities with complex MS, including consideration individual facility and measurement systems bias and precision limits. Individual facility and measurement systems bias and precision limits are required for use of such data as well as helpful in MS improvements.

For large-scale facilities such as wind tunnels and towing tanks with complex MS, only limited work done and facility or MS biases not yet considered. The NATO, AGARD, Propulsion and Energetics Panel, Uniform Engine Testing Program, was a remarkable early exercise in large-scale testing in which the same jet engines were tested in a number of jet engine test stands in various NATO countries and uncertainties were estimated to explain whether data scatter was within the data uncertainty and conclusions were drawn [10]. Ref. [11] compares results from wind tunnel tests for same geometry and conditions at two different institutes, model scales, and using a number of different measurement techniques and extensive error-analysis. The Cooperative Experimental Program of the Resistance Committees of the 17-19 International Towing Tank Conferences (ITTC) [12] compare results from towing tank tests at 22 institutes. Comparisons are made of global (resistance, sinkage and trim, wave profile, wave cut, wake survey, form factor, and blockage) and local (surface pressure and boundary layer traverses) data for a standard geometry (Series 60) of different sizes (1.2-9.6 m). However, uncertainty assessment not considered. The cooperative uncertainty assessment example for resistance test of the Resistance Committee of the 22nd ITTC [13] compare results from towing tank tests at 7 institutes of resistance test bias and precision limits and total uncertainties following standard uncertainty assessment procedures, but for different model geometries and sizes (Series 60, container ships, and 5415).

In the following, a statistical approach for estimating intervals of certification or biases of facilities or MS including uncertainties is set forth. N-order level testing reviewed followed by definitions for MxN-order level testing, which defined as M repetitions of the same N-order level experiment in M different facilities or in the same facility with M different measurement systems. If reference values known, present approach used at either the N-order or MxN-order levels. However, unlike CFD where EFD provides reference values, for EFD reference values are seldom known, e.g., from a standard facility or MS. In absence of reference values, mean facility or MS used for assessing intervals of certification or biases. Herein, certification or biases of facilities or measurement systems are defined as processes for assessing probabilistic confidence intervals for facilities or measurement systems for specific tests, data reduction equations, conditions, procedures, and uncertainty analysis. Similarly, subgroup analysis performed for isolating and assessing levels of differences due to use of different model sizes (scale effects) or measurement systems. An example provided for towing tank facilities for resistance tests using standard uncertainty analysis procedures based on an international collaboration between three facilities.

2. ESTIMATING INTERVALS OF CERTIFICATION OR BIASES OF FACILITIES OR MS

Designing tests for estimating intervals of certification or biases of facilities or MS requires special care and consideration. Many factors affect certification or biases of facilities or MS; therefore, as with estimating precision limits (i.e., random errors), only those factors specifically isolated (i.e., turned on) are included. For example, if interest is for certain types of measurements using same MS in different facilities, then model-geometry, tests, data reduction equations, conditions, procedures, and uncertainty analysis should all be the same. Ideally, standard models are used. Otherwise, effects of differences in model geometry are included. If models are geosyms, but two different scales, then scale effects are included. Similarly, if different MS used at different facilities, than effects of MS are included and so on. Although approach used for either facilities or MS, presentation that follows is for facilities since same as for the example.

2.1. N-order Level Testing

In N-order level testing, N repetitions of the same experiment in the same facility conducted

$$X_i = \frac{1}{N}\sum_{j=1}^{N} X_i^j \tag{1}$$

Where X_i^j and X_i are single realization and individual facility mean results, respectively. The uncertainty in X_i is given by the root-sum-square (RSS) of bias B_{X_i} and precision P_{X_i} limits

$$U_{X_i} = \sqrt{B_{X_i}^2 + P_{X_i}^2} \tag{2}$$

The bias limit obtained by considering all sources for systematic errors, based on 0- or 1-order testing. The precision limit obtained by the standard deviation of the mean

$$P_{X_i} = 2\frac{S_{X_i^j}}{\sqrt{N}} \tag{3}$$

Where $S_{X_i^j}$ is the standard deviation of the sample population X_i^j

$$S_{X_i^j} = \left[\frac{1}{N-1}\sum_{j=1}^{N}(X_i^j - X_i)^2\right]^{1/2} \tag{4}$$

Under the assumption of a normal distribution for the sample population X_i^j, 95% confidence level, and N≥10, the estimated true result of the experiment X_{ET_N} lies inside the intervals

$$X_i^j - U_{X_i^j} \le X_{ET_N} \le X_i^j + U_{X_i^j} \tag{5}$$

and

$$X_i - U_{X_i} \le X_{ET_N} \le X_i + U_{X_i} \tag{6}$$

for the single realization and mean experimental result, respectively, where

$$U_{X_i^j}^2 = B_{X_i}^2 + \left(2S_{X_i^j}\right)^2 \tag{7}$$

Additionally, this assumes facility biases β_{FB_i} =0.0, otherwise X_{ET_N} is the biased estimated true value. X_{ET_N} is referred to as estimated true result of the experiment; since, confidence in equations (5) and (6) relies on confidence in equations (2) and (7). Note that at the N-order level outliers often discarded if

$$\left|D_i^j\right| = \left|X_i^j - X_i\right| > 2S_{X_i^j} \tag{8}$$

2.2. *M×N-order Level Testing*

In M×N-order level testing, M repetitions of the same N-order experiment in M different facilities conducted

$$\overline{X} = \frac{1}{M}\sum_{i=1}^{M} X_i = \frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N} X_i^j \tag{9}$$

where $\overline{X}$ is the mean facility result. The uncertainty in $\overline{X}$ is

$$U_{\overline{X}} = \sqrt{B_{\overline{X}}^2 + P_{\overline{X}}^2} \tag{10}$$

The bias limit of the mean $B_{\overline{X}}$ is the average RSS of the M bias limits B_{X_i}

$$B_{\overline{X}} = \frac{1}{M}\sqrt{\sum_{i=1}^{M} B_{X_i}^2} \tag{11}$$

The precision limit $P_{\overline{X}}$ is the standard deviation of the M results X_i

$$P_{\overline{X}} = 2S_{\overline{X}} = 2\frac{S_{X_i}}{\sqrt{M}} = \frac{2}{\sqrt{M}}\left[\frac{1}{M-1}\sum_{i=1}^{M}(X_i - \overline{X})^2\right]^{1/2} \tag{12}$$

or $P_{\overline{X}}$ is the average RSS of the M precision limits P_{X_i} from equation (3)

$$P_{\overline{X}} = \frac{1}{M}\sqrt{\sum_{i=1}^{M} P_{X_i}^2} \tag{13}$$

Under the assumption of normal distribution for the sample population X_i, 95% confidence level, and M≥10, the estimated true result of the experiment $X_{ET_{MN}}$ lies inside the intervals

$$X_i - U_{X_i} \le X_{ET_{MN}} \le X_i + U_{X_i} \tag{14}$$

and

$$\overline{X} - U_{\overline{X}} \le X_{ET_{MN}} \le \overline{X} + U_{\overline{X}} \tag{15}$$

for the individual and mean facilities, respectively, where

$$U_{X_i}^2 = B_{X_i}^2 + \left(2S_{X_i}\right)^2 \tag{16}$$

Equation (16) equals equation (2) if $S_{X_i} = S_{X_i^j} / \sqrt{N}$. Additionally, this assumes the mean facility biases $\overline{\beta}_{FB}$=0.0, otherwise $X_{ET_{MN}}$ is the biased estimated true value. Fig. 1 displays M×N-order level testing, including individual and mean facility results and their bias and precision limits and total uncertainties; biased parent-population mean value μ; and estimated true experimental result $X_{ET_{MN}}$.

2.3. Using Mean Values As Reference Values

D_i defined as the difference between the N-order level individual facility X_i and MxN-order level mean facility $\overline{X}$ values

$$D_i = X_i - \overline{X} \tag{17}$$

and its uncertainty U_{D_i} is defined as the RSS of the uncertainties of X_i and $\overline{X}$

$$U_{D_i} = \sqrt{U_{X_i}^2 + U_{\overline{X}}^2} \tag{18}$$

$$U_{D_i}^2 = B_{X_i}^2 + P_{X_i}^2 + \frac{\sum B_{X_i}^2}{M^2} + \frac{4S_{X_i}^2}{M} \tag{19}$$

or

$$U_{D_i}^2 = B_{X_i}^2 + P_{X_i}^2 + \frac{\sum B_{X_i}^2}{M^2} + \frac{\sum P_{X_i}^2}{M^2} \tag{20}$$

If the absolute value of D_i is less than U_{D_i}

$$\left| D_i \right| \leq U_{D_i} \tag{21}$$

then the individual facility is certified at interval U_{D_i}, whereas if the absolute value of D_i is greater than U_{D_i}

$$\left| D_i \right| > U_{D_i} \tag{22}$$

then the facility bias U_{FB_i} is defined as

$$U_{FB_i}^2 = D_i^2 - U_{D_i}^2 \tag{23}$$

with total uncertainty

$$U_{T_i}^2 = U_{X_i}^2 + U_{FB_i}^2 \tag{24}$$

If $\left|D_i\right|$ is much greater than U_{D_i}

$$\left| D_i \right| >> U_{D_i} \tag{25}$$

then D_i approximately equals the individual facility bias error β_{FB_i}

$$D_i \cong \beta_{FB_i} \tag{26}$$

such that the biases can be estimated in both sign and magnitude and used for calibration. Fig. 2 displays use of mean values as reference values, including both situations of estimation of interval of facility certification and bias.

For certified facilities, interval certification provides additional confidence in accuracy measurements; since, validates X_i and accounts for $U_{\overline{X}}$ in assessing level of certification. For non-certified facilities, accounting for facility biases provides U_{T_i}, which is an improved estimate than U_{X_i}. Presumably, design sets the requirements on appropriate intervals for certification or biases of facilities. Comparison of U_{X_i} with $U_{\overline{X}}$ and uncertainties from other facilities is useful in developing strategies for reduction U_{X_i}. Note that for sufficiently large M, $U_{X_i} >> U_{\overline{X}}$ so $U_{D_i} \approx U_{X_i}$. In this case, M×N-order level testing primarily provides $\overline{X}$. As already mentioned, if reference values are known, present approach used at either the N-order or MxN-order levels, which for completeness is included as Appendix A.

2.4. *Subgroup Analysis*

Isolating and assessing levels of differences due to use of different model sizes (scale effects) or measurement systems is of importance. Subgroup differences assessed by comparison of the subgroup mean to the total mean, with consideration to the uncertainty in the comparison. For L subgroup facilities, subgroup mean and uncertainty given by

$$\overline{X}_{SG} = \frac{1}{L}\sum_{l=1}^{L} X_{SG_l} \tag{27}$$

$$U^2_{\overline{X}_{SG}} = B^2_{\overline{X}_{SG}} + P^2_{\overline{X}_{SG}} \tag{28}$$

where

$$B^2_{\overline{X}_{SG}} = \frac{1}{L^2}\sum_{l=1}^{L} B^2_{X_{SG_l}} \tag{29}$$

and

$$P_{\overline{X}_{SG}} = \frac{2}{\sqrt{L}}[\frac{1}{L-1}\sum_{l=1}^{L}(X_{SG_l} - \overline{X}_{SG})^2]^{1/2} \tag{30}$$

The subgroup difference and difference uncertainty given by

$$D_{SG} = \overline{X}_{SG} - \overline{X} \tag{31}$$

$$U^2_{D_{SG}} = U^2_{\overline{X}_{SG}} + U^2_{\overline{X}} \tag{32}$$

For

$$|D_{SG}| \le U_{D_{SG}} \tag{33}$$

differences can not be discerned, i.e., within noise intervals of comparison, whereas for

$$\left|D_{SG}\right| > U_{D_{SG}} \tag{34}$$

differences discernable, which suggests need for separate certification. Present approach differs from analysis of the means [14]; since, takes into account both bias and precision uncertainties for both subgroup and mean in assessing intervals of subgroup differences. Analysis of the means compares $\left|D_{SG}\right|$ with $P_{\overline{X}}\big/\sqrt{L}$.

3. EXAMPLE FOR TOWING TANK FACILITIES

It is not easy to provide an example of the proposed approach; since, as already mentioned it requires considerable resources and cooperation amongst institutes often crossing international boundaries. The example provided based on an international collaboration between three towing tank facilities for purposes of procuring benchmark CFD validation data for ship hydrodynamics resistance and propulsion geometry and conditions. Overlapping tests conducted for evaluation of facilities; measurement systems; test procedures; uncertainty assessments; model size, offsets, and turbulence stimulation; and facility/model geometry and scale effect biases. The facilities were David Taylor Model Basin (DTMB), Bethesda, MD USA; Istituto Nazionale per Studi ed Esperienze di Architettura Navale (INSEAN), Rome, Italy; and Iowa Institute of Hydraulic Research (IIHR), Iowa City, IA, USA. Hereafter designated as facilities A, B, and C, respectively. The model geometry is DTMB surface combatant 5415. Between all three facilities, many conditions and physics are under investigation. The data used as one of three test cases at the recent Gothenburg 2000 Workshop on Numerical Ship Hydrodynamics [15]. DTMB 5415 conceived by USA Navy as a preliminary design for a surface combatant ca. 1980 with a sonar dome bow and transom stern. A and B used 5.72 m, 1/24.8 scale models, whereas C used a 3.048 m, 1/46.6 scale model, as shown in Fig. 3. Thus, scale effects (subgroup analysis) considered in comparing C with A and B. The uncertainty assessment procedures closely follow [13] recommendations based on [6-7]. Ref. [16] provides an overview of the overall results of the collaboration. Although number of facilities is a minimum, the results demonstrate usefulness of approach. Since N<10, $P_{\overline{X}}$ is estimated using equation (13) as opposed to (12).

3.1. Overlapping Test Design, Data Reduction Equations, Conditions, Procedures, and Uncertainty Analysis

The most typical towing-tank tests selected for the overlapping tests, i.e., resistance, sinkage and trim, wave profile, wave elevations, and nominal wake. Each institute followed their usual procedures; however, special consideration given to integration of uncertainty assessment into all phases of the experimental process, CFD validation, and complementary CFD. Data-reduction equations defined for residuary resistance C_R, sinkage σ and trim τ, wave profile and elevations ζ, and nominal wake mean velocity $\mathbf{V}$ and pressure C_p. Similar conditions and locations were used at all three facilities: Froude number (Fr) ranges; Fr and spatial locations for uncertainty analysis; and spatial resolution for wave elevations and nominal wake.

Initial analysis of the results and attempt at identification of facility biases done by [16] using comparisons of differences between facilities (A-B, A-C, B-C) and the RSS of their uncertainties. However, this approach lacks a reference value such that the estimated facility biases depend on which facilities compared. Subsequently all results were reanalyzed according to the present approach. Presentation of the results for all tests is extensive and not necessary for the purpose of demonstration of the usefulness of the present approach. Herein, the results for the resistance test presented. The results for residuary resistance follow, whereas the results for other variables are included in Appendix B and C.

The data reduction equation for the resistance test is

$$C_R = C_T^{Tm} - C_F^{Tm}(1+k) \tag{35}$$

$$C_T^{Tm} = \frac{M_x^{Tm} g}{0.5 \rho U_c^2 S} \tag{36}$$

$$C_F = \frac{0.075}{(\log_{10} Re - 2.0)^2} \tag{37}$$

The residuary resistance C_R used since it approximately removes a portion of Reynolds number (Re) scale effects due to skin friction (but not wave breaking). C_F^{Tm} is the frictional resistance at the measured towing tank temperature Tm, k is the form factor, M_x is the force in the axial direction (resistance), ρ is the towing-tank water density, U_c is the carriage speed, and S is the design-offsets wetted surface area for the static condition. M_x

in kg is converted to Newtons by multiplication with g (g_A=9.8009; g_B=9.8033; g_C=9.8031 m/s^2) based on the local latitude. C_F and k are calculated as recommended by [17] using the model-ship correlation line equation (37) and Prohaska's method, respectively.

Tests at A were performed in basin no. 2 (575 m long, 15.5 m wide, 6.7 m deep), which is equipped with an electro-hydraulically operated drive carriage and capable of speeds of 10.3 m/s. Sidewall and end wall beaches enable 20-minute intervals between carriage runs. Washington Suburban Sanitation Commission supplied the towing-tank water. Tests at B were performed in towing tank no. 2 (220 m long, 9 m wide, 3.6 m deep), which is equipped with a single drive carriage that is capable of speeds of 10 m/s. Sidewall and end wall beaches enable 20-minute intervals between carriage runs. Natural springs supplied the towing tank water. Tests at C were performed in the IIHR towing tank (100 m long and 3.048 m wide and deep), which is equipped with an electric-motor operated drive carriage that is cable driven by a 15-horsepower motor and capable of speeds of 3 m/s. Sidewall and end wall beaches enable twelve-minute intervals between carriage runs. City of Iowa City supplied the towing tank water.

Equation (36) consists of individual MS for resistance, density, carriage speed, and surface area. Resistance measured using load cells and PC data acquisition and reduction, including statistical analysis of the sample population (average, standard deviation, minimums, maximums, outliers). Outliers identified and deleted using Chauvenet's criterion. Density is determined from measured Tm using fresh water values as recommended by ITTC Quality Manual Procedure 4.9-03-01-03 Density and Viscosity of Water. Tm is measured daily using thermometers. Carriage speed measured using encoders and PC data acquisition and reduction. Surface area is measured using templates for estimating accuracy of model offsets and rulers and weights for estimating accuracy of installation depending on whether model installed by waterline or displacement.

A used a variable reluctance, in-house manufactured load cell, signal conditioner, and 16-bit AD card. The load cell, signal conditioner, and carriage PC AD card statically calibrated on a test stand to determine the voltage-mass relationship. Data acquired by collection of 2000 discrete samples over 5 seconds at 400 Hz. Data filtered through a 10 Hz low-pass filter. B used a Hottinger Baldwin Messtechnik model U1, 50 kg load cell, signal conditioner, and 16-bit AD card. The load cell, signal conditioner, and AD card statically calibrated on a Kempf and Remmers precision test stand to determine the voltage-mass relationship. Data acquired by collection of 300 discrete samples over 10 seconds at 30 Hz. Amplified analog voltages converted to frequency (3000±2500Hz) for transmission to the AD card to reduce signal sensitivity to noise. Data filtered through a 10 Hz low-pass filter. C used a Nisshio strain-gage type 20 kg load cell, signal conditioner, and 12-bit AD card. The load cell, signal conditioner, and AD card statically calibrated on an IIHR test stand to determine the voltage-mass relationship. Data acquired by collection of 2000 discrete samples over 10 seconds at 200 Hz. Data filtered through a 3 Hz low-pass filter.

Uncertainty in C_R is equivalent to uncertainty in C_T^{Tm}; since, uncertainties in k and C_F not considered. Bias limits were estimated for individual resistance, density, carriage speed, and surface area MS, whereas precision limits were estimated end-to-end using equation (36). Table 1 summarizes the calibration, data acquisition, and data-reduction bias limits considered for each MS. Precision limits conducted over a time-period during which test conditions varied and in some cases including reinstallation of the model.

4. RESULTS

4.1. N-order level

Fig. 4 compares the individual facility (A, B, C) results for range of Fr. All three facilities conducted uncertainty analysis for Fr= 0.1, 0.28, and 0.41. Table 2 provides N-order level residuary resistance values, bias and precision limits, and total uncertainties. Trends for all three facilities are similar. Bias limits predominate for all Fr. Although not included in Table 2, bias limits for resistance and especially carriage speed are large for all speeds, whereas bias limit for surface area only significant for large speed. Precision limits increase for increasing Fr. Total uncertainty decreases for increasing Fr.

4.2 MxN-order level

In addition, Table 2 provides MxN-order level facility uncertainties and mean-facility bias and precision limits and total uncertainties and facility certification or biases. Even for M=3, $U_{X_i} > U_{\overline{X}}$ so $U_{D_i} \approx U_{X_i}$. For low speed, $D_i << U_{D_i}$ and all three facilities are certified albeit at a large interval (average 17.9%). Reduction interval of certification largely requires reduction individual facility biases for resistance and carriage speed. For medium speed, $D_i > U_{D_i}$ and all three facilities have facility biases (average 2.3%) and total uncertainties larger than individual facility estimates, especially for C. For high speed, $D_i > U_{D_i}$ and facility biases and total uncertainties for B and C are large (uncertainty analysis not available for A). Fig 4 and Table 2 suggest scale effects important for C, for medium and high speed, as shown next using subgroup analysis.

4.3 Subgroup Analysis

Table 3 is similar to Table 2, but for subgroup analysis in order to isolate scale effects due to smaller model size used at C. In this case mean facility based only on facilities A and B, which used same model size, and subgroup analysis based on facility C. For low speed, scale effects not discernable and facility certification similar as before. For medium and high speed, conclusions different from before, i.e., for medium speed facilities A and B certified at about 2% interval, whereas facility C has nearly 8% interval facility bias. For high speed, facility B has small and facility C large facility biases.

5 SUMMARY AND CONCLUSIONS

A statistical approach is set forth for assessing probabilistic confidence intervals (i.e., intervals of certification or biases of facilities of MS) for facilities or MS for specific tests, data reduction equations, conditions, procedures, and uncertainty analysis based on MxN-order level testing and use of mean facility or MS as reference values. MxN-order level testing defined as M repetitions of the same N-order level experiment in M different facilities or in the same facility with M different measurement systems. Similarly, subgroup analysis performed for isolating and assessing levels of differences due to use of different model sizes (scale effects) or measurement systems.

An example provided for towing tank facilities for resistance tests using standard uncertainty analysis procedures based on an international collaboration between three facilities: two using larger models and one using smaller model. Although the number of facilities is a minimum, the results demonstrate usefulness of approach, including subgroup analysis for isolating differences due to use of different model sizes. For low speed, all three facilities are certified, but at a large interval (average 17.9%). Reduction interval of certification largely requires reduction individual facility biases for resistance and carriage speed. For medium speed, facilities with larger models certified at about 2%, whereas facility with smaller model shows 7.2% facility bias. For high speed, facilities with larger and smaller models show 2.9% and 9.3% facility biases, respectively. For certified facilities, interval certification provides additional confidence in accuracy measurements. For non-certified facilities, accounting for facility biases provides improved individual facility uncertainties. Presumably, design sets the requirements on appropriate intervals for certification or biases of facilities. The results are also useful for developing strategies for reduction intervals of individual facility uncertainties and certification or facility biases.

It is reasonable to expect that results from more facilities for same model geometry with uncertainty analysis will in fact provide confirmation of normal distributions for MxN-order level testing and improved estimates for intervals of certification or biases of facilities. This based on previous work between more facilities using same model geometry but without uncertainty analysis and using uncertainty analysis but with different model geometry. It is also reasonable to expect that situation will be similar for different types of facilities and MS. Both these expectations support recommendation of future collaborations between more facilities. International collaborations are attractive from resource perspective and in achieving ground truth. Knowledge of intervals of certification or biases is important for design, accrediting facilities or measurement systems, and CFD validation.

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TABLE AND FIGURES

Table 1 Bias limits for resistance, density, carriage speed, and surface area

	Calibration	Data Acquisition	Data Reduction
Resistance	Load Cell	Curve fit	Alignment load cell
Density	Thermometer		
Carriage Speed	Encoder Wheel diameter Time base	AD conversions Curve fit	
Surface area	Template Rulers Weights		

Table 2. N-order level residuary resistance values, bias and precision limits, and total uncertainties; MxN-order level facility uncertainties and mean-facility bias and precision limits and total uncertainties; and facility certification or biases.

Fr & Facility		**N-order level (% X_i)**				**M×N-order level (% $\bar{X}$)**				**Facility Certification or biases (% $\bar{X}$)**			
		X_i	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	6.00E-04	76.3%	23.7%	10.4%	11.3%				8.6%	14.5%	0	11.3%
	B	5.23E-04	69.4%	30.6%	21.2%	20.1%	75.7%	24.3%	9.1%	-5.3%	22.0%	0	20.1%
	C	5.34E-04	87.6%	12.4%	14.9%	14.5%				-3.3%	17.1%	0	14.5%
	AVE	5.52E-04	77.8%	22.2%	15.5%	15.3%				0.0%	17.9%	0	15.3%
0.28	A	1.33E-03	45.5%	54.5%	1.1%	1.1%				3.0%	1.6%	2.6%	2.8%
	B	1.32E-03	80.0%	20.0%	2.1%	2.1%	81.6%	18.4%	1.2%	2.2%	2.4%	0	2.1%
	C	1.22E-03	89.2%	10.8%	2.7%	2.6%				-5.3%	2.8%	4.4%	5.1%
	AVE	1.29E-03	71.6%	28.4%	2.0%	1.9%				0.0%	2.3%	2.3%	3.4%
0.41	A	3.69E-03	NA	NA	NA	NA				-0.2%	NA	NA	NA
	B	3.94E-03	66.3%	33.7%	1.1%	1.2%	73.6%	26.4%	0.8%	6.7%	1.4%	6.5%	6.6%
	C	3.45E-03	80.5%	19.5%	1.3%	1.2%				-6.5%	1.4%	6.3%	6.5%
	AVE	3.69E-03	73.4%	26.6%	1.2%	1.2%				0.0%	1.4%	6.4%	6.5%

Table 3. Subgroup Analysis: N-order level residuary resistance values, bias and precision limits, and total uncertainties; MxN-order level facility uncertainties and mean facility bias and precision limits and total uncertainties; and facility certification or biases

Fr &	Facility	N-order level (% X_i)				M×N-order level (% $\bar{X}$)				Facility Certification or biases (% $\bar{X}$)			
		X_i	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	6.00E-04	76.3%	23.7%	10.4%	11.1%				6.8%	15.9%	0	11.1%
	B	5.23E-04	69.4%	30.6%	21.2%	19.8%	71.1%	28.9%	11.3%	-6.8%	22.8%	0	19.8%
	C	5.34E-04	87.6%	12.4%	14.9%	14.2%				-4.8%	18.2%	0	14.2%
	AVE	5.61E-04	77.8%	22.2%	15.5%	15.0%				-1.6%	18.9%	0	15.0%
0.28	A	1.33E-03	45.5%	54.5%	1.1%	1.1%				0.4%	1.6%	0	1.1%
	B	1.32E-03	80.0%	20.0%	2.1%	2.1%	72.8%	27.2%	1.2%	-0.4%	2.4%	0	2.1%
	C	1.22E-03	89.2%	10.8%	2.7%	2.5%				-7.7%	2.8%	7.2%	7.6%
	AVE	1.33E-03	71.6%	28.4%	2.0%	1.9%				-2.6%	2.3%	2.4%	3.6%
0.41	A	3.69E-03	NA	NA	NA	NA				-3.3%	NA	NA	NA
	B	3.94E-03	66.3%	33.7%	1.1%	1.1%	66.3%	33.7%	1.1%	3.3%	1.6%	2.9%	3.1%
	C	3.45E-03	80.5%	19.5%	1.3%	1.2%				-9.5%	1.6%	9.3%	9.4%
	AVE	3.81E-03	73.4%	26.6%	1.2%	1.1%				-3.2%	1.6%	6.1%	6.3%

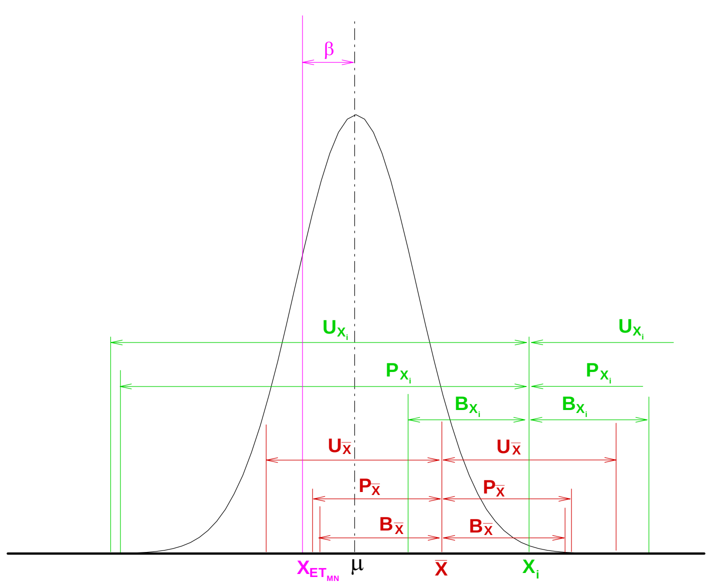


Figure 1. M×N-order level testing

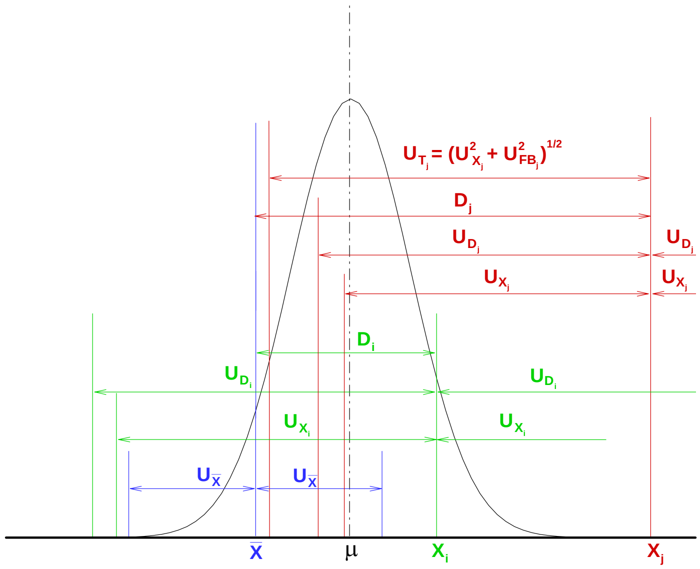


Fig 2 Estimating intervals of certification or biases of facilities using mean as reference value: facility i certified at U_{D_i} and facility j with facility bias U_{FB_j}.

(a) DTMB model 5415

(b) INSEAN model 2340A

(c) IIHR model 5512

Fig 3 Surface combatant model ships

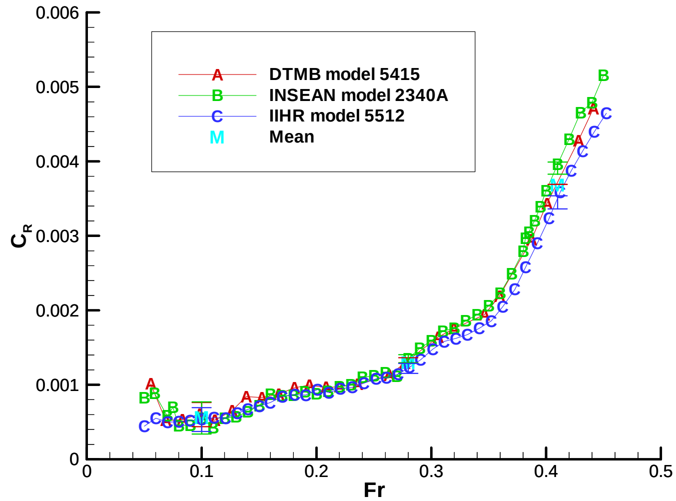


Figure 4 Individual facility residuary resistance results and mean facility residuary resistance and individual facility uncertainty bands at Fr=0.1, 0.28 and 0.41.

APPENDIX A: ESTIMATING INTERVALS OF CERTIFICATION OR BIASES OF FACILITIES OR MS USING REFERENCE VALUES

Assume a reference value known for the experimental result designated X_R with uncertainty U_R both of which considered as standard values. Although U_R is likely much less than U_{X_i} or $U_{\overline{X}}$, it is retained for completeness.

For the mean facility, $\overline{D}$ is defined as the difference between the mean facility $\overline{X}$ and reference X_R values

$$\overline{D} = \overline{X} - X_R \tag{A1}$$

and its uncertainty $U_{\overline{D}}$ is defined as the RSS of the uncertainties of $\overline{X}$ and X_R

$$U_{\overline{D}} = \sqrt{U_{\overline{X}}^2 + U_R^2} \tag{A2}$$

If the absolute value of $\overline{D}$ is less than $U_{\overline{D}}$

$$\left|\overline{D}\right| \leq U_{\overline{D}} \tag{A3}$$

then the mean facility is certified at the interval $U_{\overline{D}}$, whereas if the absolute value of $\overline{D}$ is greater than $U_{\overline{D}}$

$$\left|\overline{D}\right| > U_{\overline{D}} \tag{A5}$$

then the mean facility bias $U_{\overline{FB}}$ is defined as

$$U_{\overline{FB}}^2 = \overline{D}^2 - U_{\overline{D}}^2 \tag{A6}$$

with total uncertainty $U_{\overline{T}}$

$$U_{\overline{T}}^2 = U_{\overline{X}}^2 + U_{\overline{FB}}^2 \tag{A7}$$

If $\left|\overline{D}\right|$ is much greater than $U_{\overline{D}}$

$$\left|\overline{D}\right| >> U_{\overline{D}} \tag{A8}$$

then $\overline{D}$ approximately equals the mean facility bias error $\overline{\beta}_{FB}$

$$\overline{D} \cong \overline{\beta}_{FB} \tag{A9}$$

such that the biases can be estimated in both sign and magnitude and used for calibration.

For the individual facility, D_i is defined as the difference between the individual facility X_i and reference X_R values

$$D_i = X_i - X_R \tag{A10}$$

and its uncertainty U_{D_i} is defined as the RSS of the uncertainties of X_i and X_R

$$U_{D_i} = \sqrt{U_{X_i}^2 + U_R^2} \tag{A11}$$

If the absolute value of D_i is less than U_{D_i}

$$|D_i| \le U_{D_i} \tag{A12}$$

then the individual facility is certified at the interval U_{D_i}, whereas if the absolute value of D_i is greater than U_{D_i}

$$|D_i| > U_{D_i} \tag{A13}$$

then the individual facility bias U_{FB_i} is defined as

$$U_{FB_i}^2 = D_i^2 - U_{D_i}^2 \tag{A14}$$

with total uncertainty U_{T_i}

$$U_{T_i}^2 = U_{X_i}^2 + U_{FB_i}^2 \tag{A15}$$

If $|D_i|$ is much greater than U_{D_i}

$$|D_i| >> U_{D_i} \tag{A16}$$

then D_i approximately equals the individual facility bias error β_{FB_i}

$$D_i \cong \beta_{FB_i} \tag{A17}$$

such that the biases can be estimated in both sign and magnitude and used for calibration.

APPENDIX B: FACILITY CERTIFICATION/BIASES FOR SINKAGE, TRIM, WAVE PROFILE, WAVE ELEVATION, AND NOMINAL WAKE VELOCITY USING AVERAGE $\bar{X}$ = (A+B+C)/3

Note for Table 2 in text:

(1) X_i for A and C from Interpolated data files (dtmb_intp.tec, iihr_intp.tec)

(2) X_i for B from IIHR Report 421 (p18) for INSEAN

(3) All $B_{X_i}^2$ and $P_{X_i}^2$ from [16]

(4) U_{X_i} for A from DTMB’s Raw data files (dtmb_resist.tec)

(5) U_{X_i} for B from Angelo Olivieri’s email to Fred Stern (04/08/2003)

(6) U_{X_i} for C from IIHR Raw data document

Table B1. Facility certification or biases for sinkage results σ

Fr & Facility		N-order level (% X_i)				M×N-order level (% $\overline{X}$)				Facility Certification or biases (% $\overline{X}$)			
		X_i	$B_{X_i}^2$	$P_{X_i}^2$	U_{X_i}	U_{X_i}	$B_{\overline{X}}^2$	$P_{\overline{X}}^2$	$U_{\overline{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	7.49E-04	75.6%	24.4%	12.2%	13.4%				10.1%	20.3%	0	13.4%
	B	6.98E-04	0.0%	100.0%	42.0%	43.1%	8.8%	91.2%	15.3%	2.6%	45.7%	0	43.1%
	C	5.93E-04	82.2%	17.8%	8.7%	7.6%				-12.8%	17.1%	0	7.6%
	AVE	6.80E-04	52.6%	47.4%	21.0%	21.4%				0.0%	27.7%	0	21.4%
0.28	A	7.35E-03	68.4%	32.6%	5.6%	5.5%				-1.0%	6.1%	0	5.5%
	B	7.39E-03	0.0%	100.0%	4.7%	4.7%	39.3%	60.7%	2.5%	-0.3%	5.3%	0	4.7%
	C	7.51E-03	30.4%	69.6%	1.4%	1.4%				1.3%	2.9%	0	1.4%
	AVE	7.42E-03	32.9%	67.4%	3.9%	3.9%				0.0%	4.8%	0	3.9%
0.41	A	1.73E-02	56.3%	44.7%	2.5%	2.4%				-5.1%	2.7%	4.3%	4.9%
	B	1.88E-02	0.0%	100.0%	2.9%	3.0%	21.9%	78.1%	1.3%	3.3%	3.3%	0	3.0%
	C	1.85E-02	42.8%	57.2%	0.6%	0.6%				1.8%	1.4%	1.1%	1.2%
	AVE	1.82E-02	33.0%	67.3%	2.0%	2.0%				0.0%	2.5%	1.8%	3.1%

Note:

(1) DRE for sinkage in ONR paper figures for three institutes is

$$\sigma = \frac{2(\Delta FP + \Delta AP)}{L} \text{ instead of } \sigma = \frac{2}{Fr^2}\frac{2(\Delta FP + \Delta AP)}{2L}$$

(2) Uncertainties in terms of percentages remain the same for both above DREs, while uncertainty magnitudes are different as long as we assume that Fr number has no uncertainty for simplicity.

(3) X_i for A, B, and C from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)

(4) All $B_{X_i}^2$ and $P_{X_i}^2$ from [16]

(5) $B_{X_i}^2$ and $P_{X_i}^2$ for DTMB (Fr=0.28 and 0.41) are best estimates from [16]

(6) All U_{X_i} based on mean values from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)

Table B2. Facility certification or biases for trim results τ

Fr & Facility		N-order level (% X_i)				M×N-order level (% $\bar{X}$)				Facility Certification or biases (% $\bar{X}$)			
		X_i	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	-2.05E-04	64.5%	35.5%	14.4%	6.1%				-57.4%	15.7%	55.2%	55.5%
	B	-6.13E-04	0.0%	100.0%	32.0%	40.9%	6.0%	94.0%	14.5%	27.7%	43.3%	0	40.9%
	C	-6.22E-04	50.8%	49.2%	10.2%	13.3%				29.7%	19.6%	22.3%	25.9%
	AVE	-4.80E-04	38.4%	61.6%	18.9%	20.1%				0.0%	26.2%	25.8%	40.8%
0.28	A	-3.90E-03	54.7%	46.3%	2.8%	2.6%				-5.8%	3.2%	4.8%	5.5%
	B	-3.77E-03	0.0%	100.0%	4.7%	4.3%	18.1%	81.9%	1.8%	-8.9%	4.7%	7.6%	8.8%
	C	-4.75E-03	36.1%	63.9%	1.8%	2.1%				14.7%	2.8%	14.5%	14.6%
	AVE	-4.14E-03	30.3%	70.1%	3.1%	3.0%				0.0%	3.5%	9.0%	9.6%
0.41	A	1.36E-02	38.1%	61.9%	1.5%	1.6%				6.6%	1.8%	6.4%	6.6%
	B	1.40E-02	0.0%	100.0%	0.9%	1.0%	18.9%	81.1%	0.8%	10.1%	1.2%	10.0%	10.0%
	C	1.06E-02	4.1%	95.9%	1.8%	1.5%				-16.7%	1.7%	16.6%	16.6%
	AVE	1.27E-02	14.1%	85.9%	1.4%	1.3%				0.0%	1.6%	11.0%	11.1%

Note:

(1) DRE for trim in ONR paper figures for three institutes is

$$\tau = \frac{2(\Delta AP - \Delta FP)}{L} \text{ instead of } \tau = \frac{2}{Fr^2}\frac{(\Delta AP - \Delta FP)}{L}$$

(2) Uncertainties in terms of percentages remain the same for both above DREs, while uncertainty magnitudes are different as long as we assume that Fr number has no uncertainty for simplicity.

(3) X_i for A, B, and C from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)

(4) All $B^2_{X_i}$ and $P^2_{X_i}$ from [16]

(5) $B^2_{X_i}$ and $P^2_{X_i}$ for DTMB (Fr=0.28) are best estimates from [16]

(6) All U_{X_i} based on mean values from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)

Table B3. Facility certification or biases for wave profile results ζ

Fr & Facility		N-order level (% D_{X_i})					M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
		X_i	D_{X_i}	$B_{X_i}^2$	$P_{X_i}^2$	U_{X_i}	U_{X_i}	$B_{\bar{X}}^2$	$P_{\bar{X}}^2$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	1.82E-02	1.89E-02	64.5%	35.5%	3.5%	3.2%	85.9%	14.1%	2.2%	-6.4%	3.9%	5.1%	6.0%
	B	2.00E-02	2.09E-02	100.0%	0.0%	4.2%	4.2%				2.4%	4.7%	0	4.2%
	C	2.03E-02	2.22E-02	83.7%	16.3%	3.4%	3.7%				3.9%	4.3%	0	3.7%
	AVE	1.95E-02	2.07E-02	82.7%	17.3%	3.7%	3.7%				0.0%	4.3%	1.7%	4.6%
0.41	A	3.02E-02	3.62E-02	64.5%	35.5%	1.8%	2.0%	81.5%	18.5%	1.2%	2.0%	2.3%	0	2.0%
	B	2.82E-02	2.46E-02	100.0%	0.0%	2.6%	1.9%				-4.2%	2.3%	3.5%	4.0%
	C	3.03E-02	3.86E-02	81.6%	18.4%	2.0%	2.3%				2.2%	2.6%	0	2.3%
	AVE	2.95E-02	3.32E-02	82.0%	18.0%	2.1%	2.1%				0.0%	2.4%	1.2%	2.8%

X_i **is the maximum elevation on the wave profile.**

Note:

(1) X_i from Data files before interpolation (dtmb_dpwpro28.tec, sean_dpwpro28.tec, iihr_dpwpro28.tec)

(2) D_{X_i} for A from DTMB's website: http://www50.dt.navy.mil/5415/profile.html, (0.15 inch for all points)

(3) D_{X_i} for B from IIHR Report 421 (p.53) for INSEAN

(4) D_{X_i} for C from IIHR's raw data files (wpro2801.tec, wpro2802.tec, wpro2803.tec, wpro4101.tec, wpro4102.tec, wpro4103.tec)

(5) All $B_{X_i}^2$ and $P_{X_i}^2$ from [16]

(6) All U_{X_i} from [16]

Table B4. Facility certification or biases for wave elevation results ζ at cut y=0.324

Fr & Facility		N-order level (% D_{X_i})					M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
		X_i	D_{X_i}	$B_{X_i}^2$	$P_{X_i}^2$	U_{X_i}	U_{X_i}	$B_{\bar{X}}^2$	$P_{\bar{X}}^2$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	4.19E-03	1.17E-02	76.6%	23.4%	2.7%	2.8%				-1.8%	3.3%	0	2.8%
	B	5.20E-03	1.09E-02	64.9%	35.1%	2.4%	2.3%	66.0%	34.0%	1.7%	7.2%	2.9%	6.6%	7.0%
	C	3.79E-03	1.11E-02	59.0%	41.0%	3.4%	3.4%				-5.4%	3.8%	3.9%	5.1%
	AVE	4.39E-03	1.12E-02	66.8%	33.2%	2.9%	2.9%				0.0%	3.3%	3.5%	5.0%

X_i **is the maximum elevation on the cut y=0.324.**

Note:

(1) X_i from Interpolated data files (dtmb_int324.tec, insean_int324.tec, iihr_int324.tec)

(2) D_{X_i} for A from Interpolated data files (dtmb_intpat.tec)

(3) D_{X_i} for B from IIHR Report 421 (p.54) for INSEAN

(4) D_{X_i} for C from (Joe Longo) IIHR's raw data file (a0_101.dat zone = "Steady")

(5) All $B_{X_i}^2$ and $P_{X_i}^2$ from [16]

(6) All U_{X_i} from [16]

(7) $B_{X_i}^2$ and $P_{X_i}^2$ for DTMB (Fr=0.28) are best estimates from [16]

Table B5. Facility certification or biases for wave elevation results ζ

Fr & Facility		N-order level (% D_{X_i})				M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
		D_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	1.17E-02	76.6%	23.4%	2.7%	2.8%	66.0%	34.0%	1.7%	4.1%	3.3%	2.5%	3.8%
	B	1.09E-02	64.9%	35.1%	2.4%	2.3%				2.8%	2.9%	0	2.3%
	C	1.11E-02	59.0%	41.0%	3.4%	3.4%				2.0%	3.8%	0	3.4%
	AVE	1.12E-02	66.8%	33.2%	2.9%	2.9%				2.9%	3.3%	0.8%	3.2%

Note:

(1) D_{X_i} for A from Interpolated data files (dtmb_intpat.tec)

(2) D_{X_i} for B from IIHR Report 421 (p.54) for INSEAN

(3) D_{X_i} for C from (Joe Longo) IIHR's raw data files (a0_101.dat zone = "Steady")

(4) All $B^2_{X_i}$ and $P^2_{X_i}$ from [16]

(5) All U_{X_i} from [16]

(6) $B^2_{X_i}$ and $P^2_{X_i}$ for DTMB (Fr=0.28) are best estimates from [16]

(7)All D_i calculated from Interpolated data files (dtmb_intpat.tec, insean_intpat.tec, iihr_intpat.tec)

Table B6. Facility certification or biases for nominal wake velocity results at Fr=0.28

Fr & Facility		N-order level (% D_{X_i})										M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
			LTR			HTR			SQRT (LTR^2+HTR^2)/2										
		D_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28 U	A	1.00E+00	65.4%	34.4%	1.6%	74.5%	26.5%	12.5%	73.6%	26.4%	6.3%	6.3%	75.3%	24.7%	2.2%	1.5%	6.6%	0	6.3%
	B	1.02E+00	47.8%	52.2%	0.4%	60.4%	39.6%	0.4%	53.5%	46.5%	0.3%	0.3%				1.3%	2.2%	0	0.3%
	C	1.01E+00	99.8%	0.2%	1.2%	99.2%	0.8%	3.1%	99.3%	0.7%	1.7%	1.7%				1.7%	2.7%	0	1.7%
	AVE	1.01E+00	71.0%	28.9%	1.1%	78.0%	22.3%	5.3%	75.5%	24.5%	2.8%	2.7%				1.5%	3.9%	0	2.7%
0.28 V	A	9.62E-02	54.3%	46.7%	2.9%	43.5%	56.5%	6.5%	45.2%	54.8%	3.6%	3.0%	71.6%	28.4%	1.7%	4.1%	3.4%	2.2%	3.7%
	B	1.19E-01	21.2%	78.8%	1.9%	15.9%	84.1%	2.7%	17.6%	82.4%	1.6%	1.7%				4.3%	2.4%	3.6%	4.0%
	C	1.33E-01	99.7%	0.3%	5.5%	93.3%	6.7%	3.8%	97.7%	2.3%	3.3%	3.8%				4.0%	4.2%	0	3.8%
	AVE	1.16E-01	58.4%	41.9%	3.4%	50.9%	49.1%	4.3%	53.5%	46.5%	2.9%	2.8%				4.1%	3.3%	1.9%	3.8%
0.28 W	A	1.47E-01	65.3%	34.7%	6.5%	44.6%	35.4%	3.7%	63.3%	36.7%	3.6%	3.3%	81.7%	18.3%	1.6%	6.0%	3.6%	4.8%	5.8%
	B	1.65E-01	79.1%	20.9%	1.0%	87.9%	12.1%	0.9%	83.1%	16.9%	0.6%	0.6%				11.3%	1.7%	11.2%	11.2%
	C	1.79E-01	99.9%	0.1%	4.1%	99.2%	0.8%	4.5%	99.5%	0.5%	3.0%	3.3%				5.8%	3.7%	4.5%	5.6%
	AVE	1.64E-01	81.4%	18.6%	3.8%	77.2%	16.1%	3.0%	82.0%	18.0%	2.4%	2.4%				7.7%	3.0%	6.8%	7.5%

Note:

(1) D_{X_i} for A from DTMB Raw data files: (dtmb_dpnwake.tec)

(2) D_{X_i} for B from IIHR Report 421 (p.56) for INSEAN, but dynamic range for U is divided by 2.

(3) D_{X_i} for C from IIHR Raw data document

(4) All $B^2_{X_i}$ and $P^2_{X_i}$ from [16]

(5) All U_{X_i} from [16]

(6) $B_{X_i}^2$ and $P_{X_i}^2$ for DTMB are best estimates from [16]

(7) All D_i calculated from Interpolated data files (dtmb_dpINTnwake.tec, sean_dpINTnwake.tec, iihr_dpINTnwake.tec)

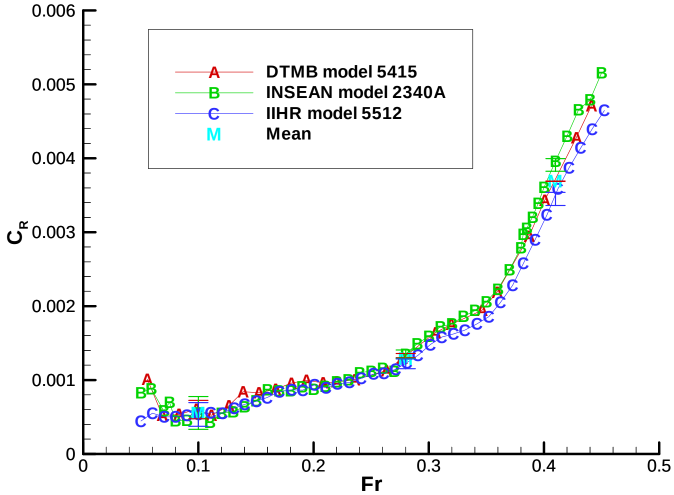


Figure B1 Residuary resistance results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Resistance\dtmb_dpcr.tec

(2) INSEAN data file: \23rdONRdata\Resistance\sean_dpcr.tec

(3) IIHR data file: \23rdONRdata\Resistance\iihr_dpcr.tec

(4) Mean data file: \23rdONRdata\Resistance\meanABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Resistance\dtmb_dpcr_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Resistance\sean_dpcr_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Resistance\iihr_dpcr_un.dat

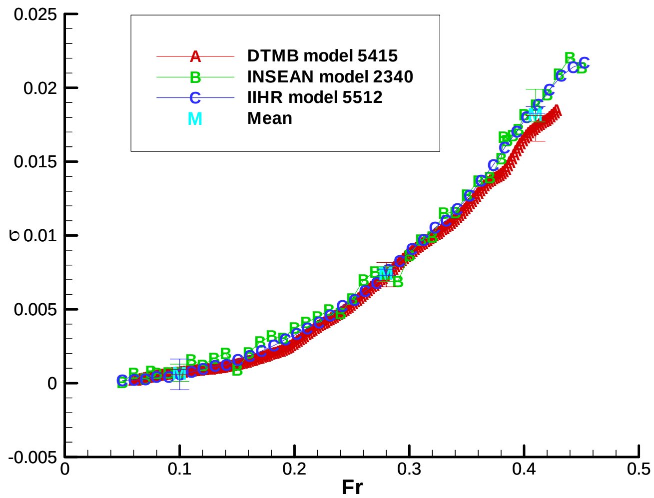


Figure B2 Sinkage results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Sinkage_Trim\dtmb_dpst.tec

(2) INSEAN data file: \23rdONRdata\Sinkage_Trim\sean_dpst.tec

(3) IIHR data file: \23rdONRdata\Sinkage_Trim\iihr_dpst.tec

(4) Mean data file: \23rdONRdata\Sinkage_Trim\meanABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Sinkage_Trim\dtmb_dpcr_sink_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Sinkage_Trim\sean_dpcr_sink_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Sinkage_Trim\iihr_dpcr_sink_un.dat

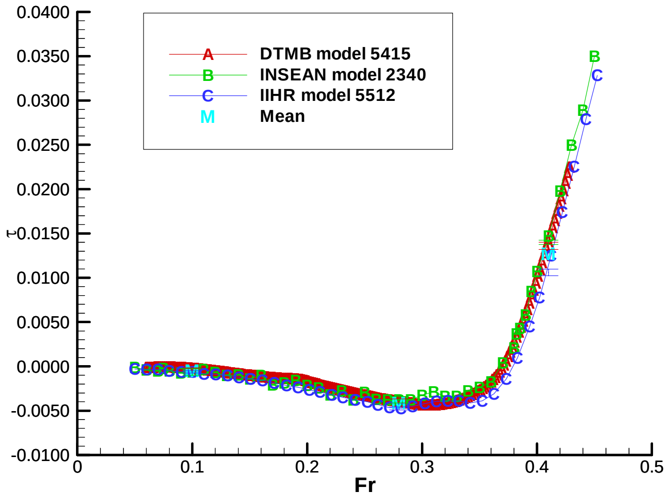


Figure B3 Trim results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Sinkage_Trim\dtmb_dpst.tec

(2) INSEAN data file: \23rdONRdata\Sinkage_Trim\sean_dpst.tec

(3) IIHR data file: \23rdONRdata\Sinkage_Trim\iihr_dpst.tec

(4) Mean data file: \23rdONRdata\Sinkage_Trim\meanABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Sinkage_Trim\dtmb_dpcr_trim_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Sinkage_Trim\sean_dpcr_trim_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Sinkage_Trim\iihr_dpcr_trim_un.dat

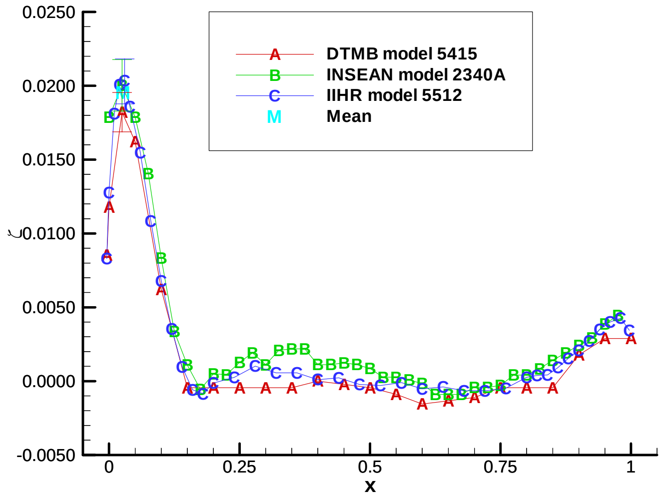


Figure B4 Wave profile results and uncertainty bands at ζ_{max} (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Profiles\dtmb_dpwpro28.tec

(2) INSEAN data file: \23rdONRdata\Wave_Profiles\sean_dpswpro28.tec

(3) IIHR data file: \23rdONRdata\Wave_Profiles\iihr_dpwpro28.tec

(4) Mean data file: \23rdONRdata\Wave_Profiles\mean28ABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Profiles\dtmb_dp28_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Profiles\sean_dp28_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Profiles\iihr_dp28_un.dat

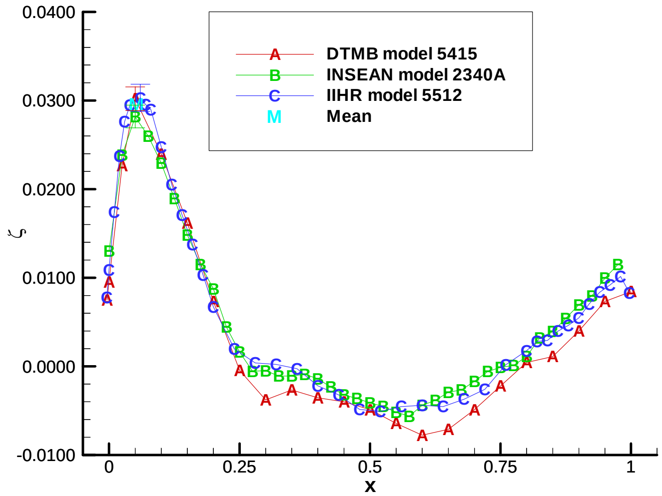


Figure B5 Wave profile results and uncertainty bands at ζ_{max} (Fr=0.41)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Profiles\dtmb_dpwpro41.tec

(2) INSEAN data file: \23rdONRdata\Wave_Profiles\sean_dpswpro41.tec

(3) IIHR data file: \23rdONRdata\Wave_Profiles\iihr_dpwpro41.tec

(4) Mean data file: \23rdONRdata\Wave_Profiles\mean41ABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Profiles\dtmb_dp41_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Profiles\sean_dp41_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Profiles\iihr_dp41_un.dat

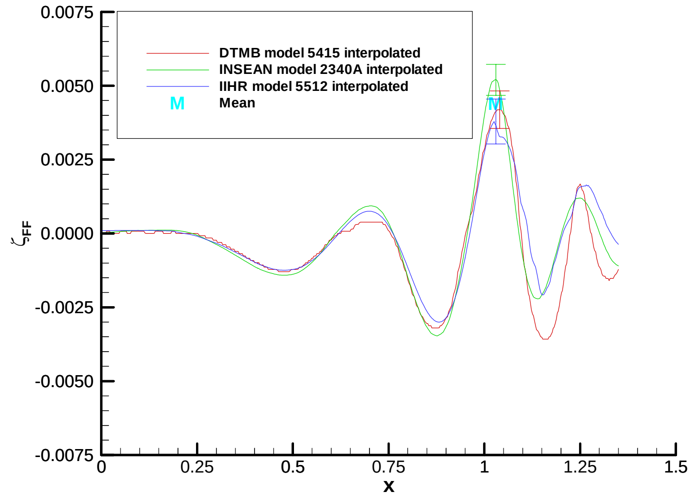


Figure B6 Wave elevation results and uncertainty bands for cut y=0.324 (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\dtmb_int324.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\sean_int324.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\iihr_int324.tec

(4) Mean data file: \23rdONRdata\Wave_Elevations\meanABC.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Elevations\dtmb_dp28_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Elevations\sean_dp28_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Elevations\iihr_dp28_un.dat

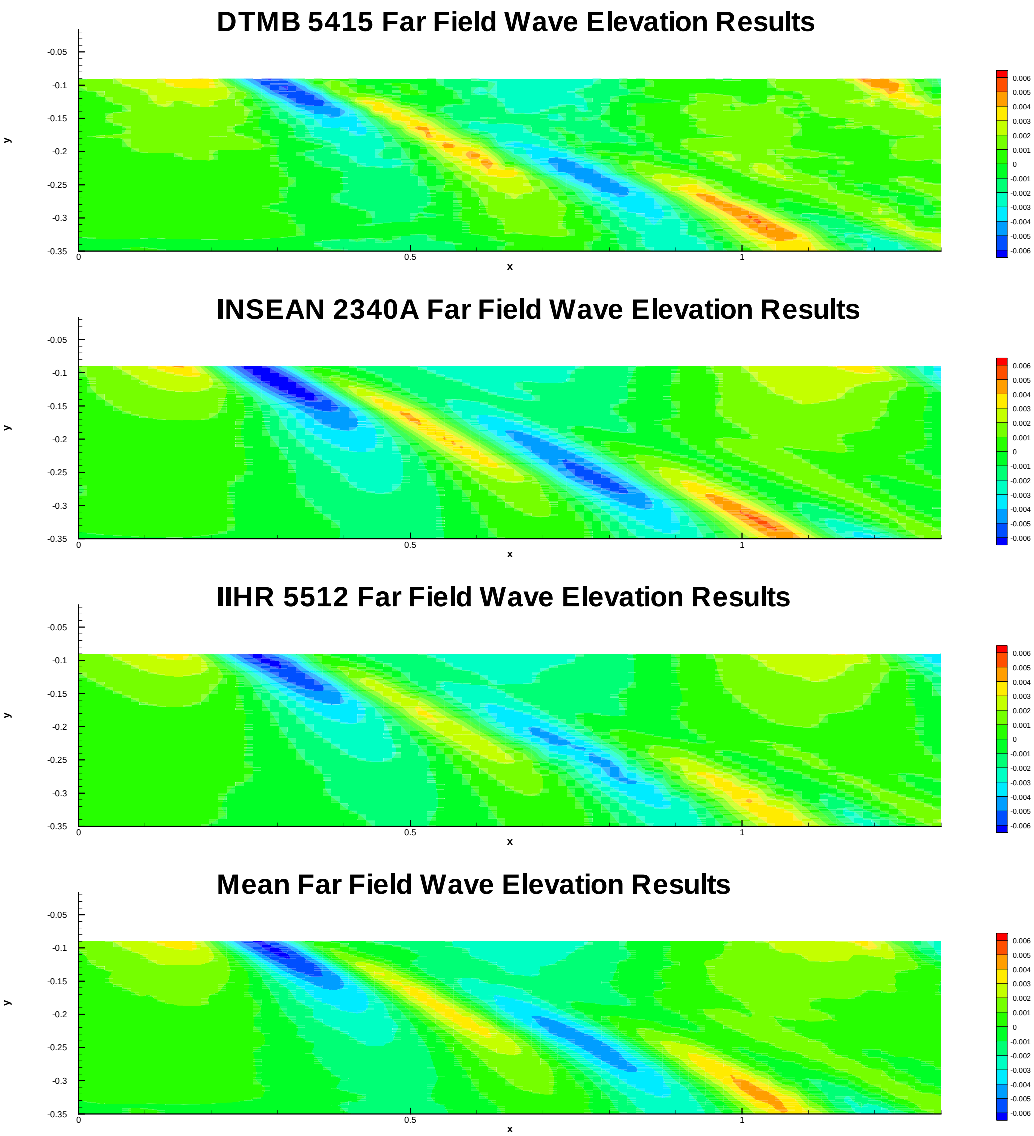


Figure B7 Wave elevation results for far field and the mean result (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\dtmb_intpat.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\insean_intpat.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\iihr_intpat.tec

(4) Mean data file: \23rdONRdata\Wave_Elevations\mean_Zeta_ABC.dat

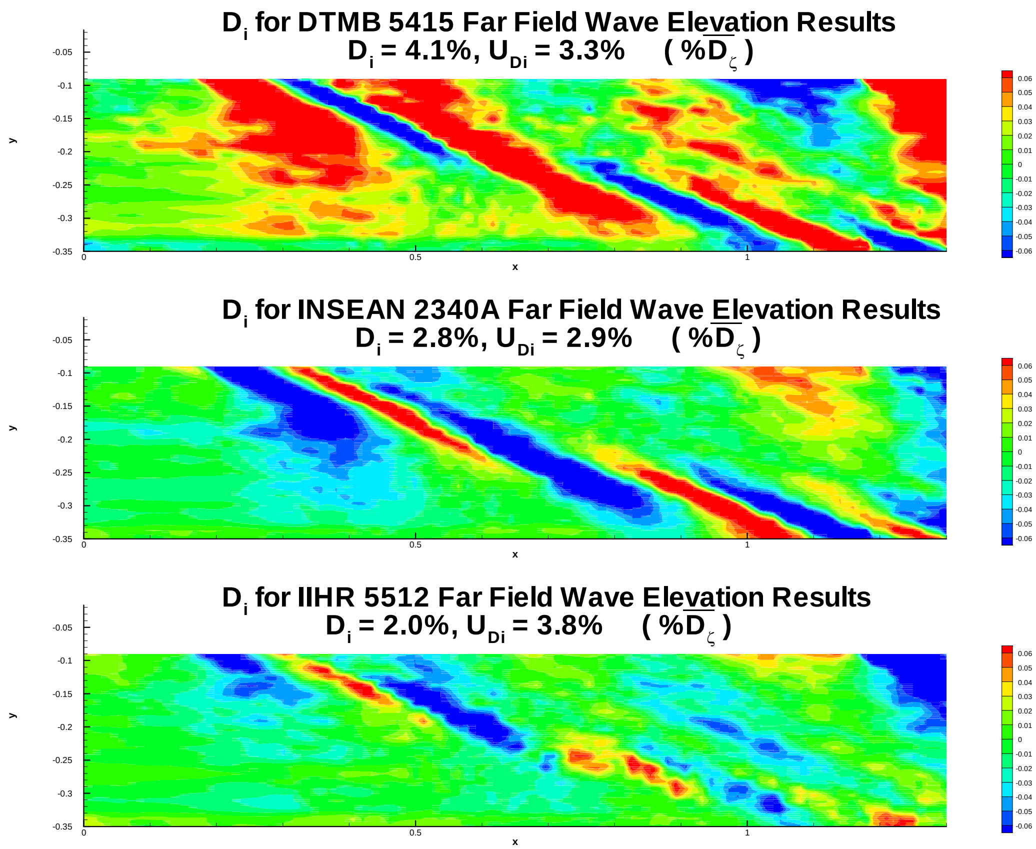


Figure B8 Contours of difference Di of wave elevation results (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\DTMB_DZeta_ABC.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\SEAN_DZeta_ABC.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\IIHR_DZeta_ABC.tec

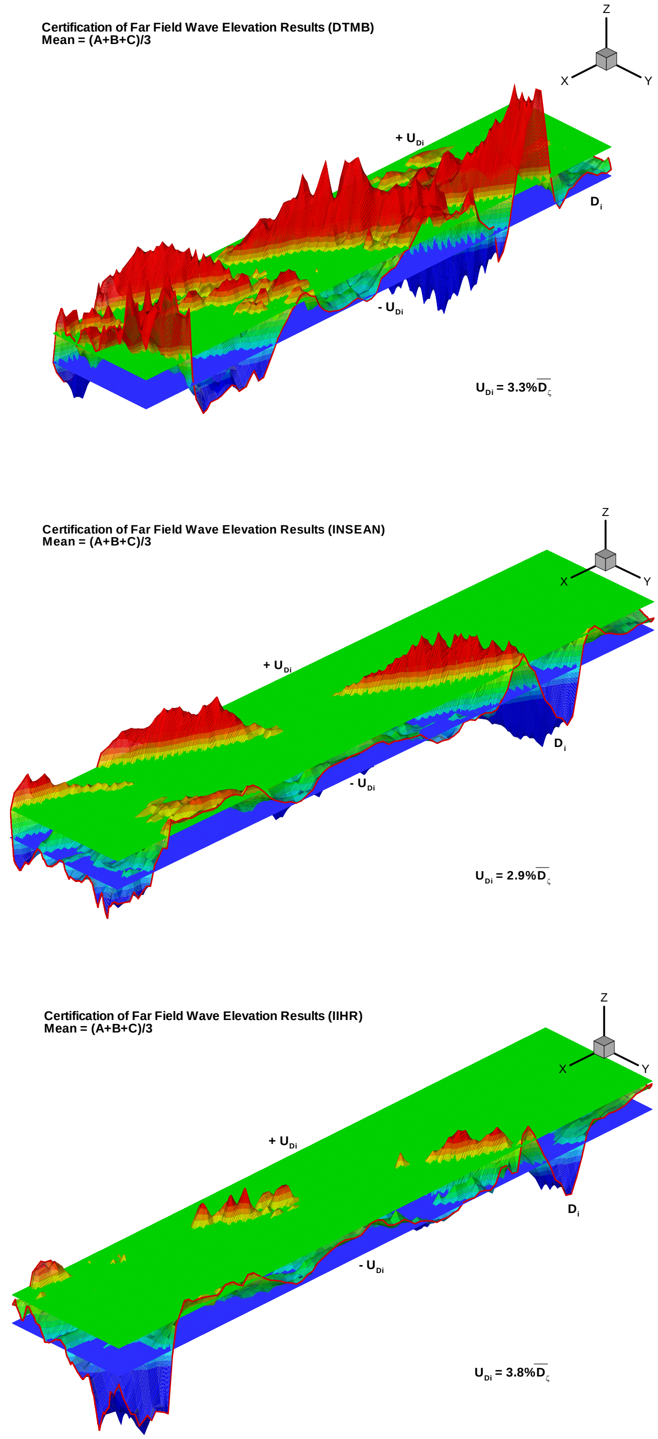


Figure B9 3D surface of difference Di of wave elevation results (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\DTMB_DZeta_ABC.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\SEAN_DZeta_ABC.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\IIHR_DZeta_ABC.tec

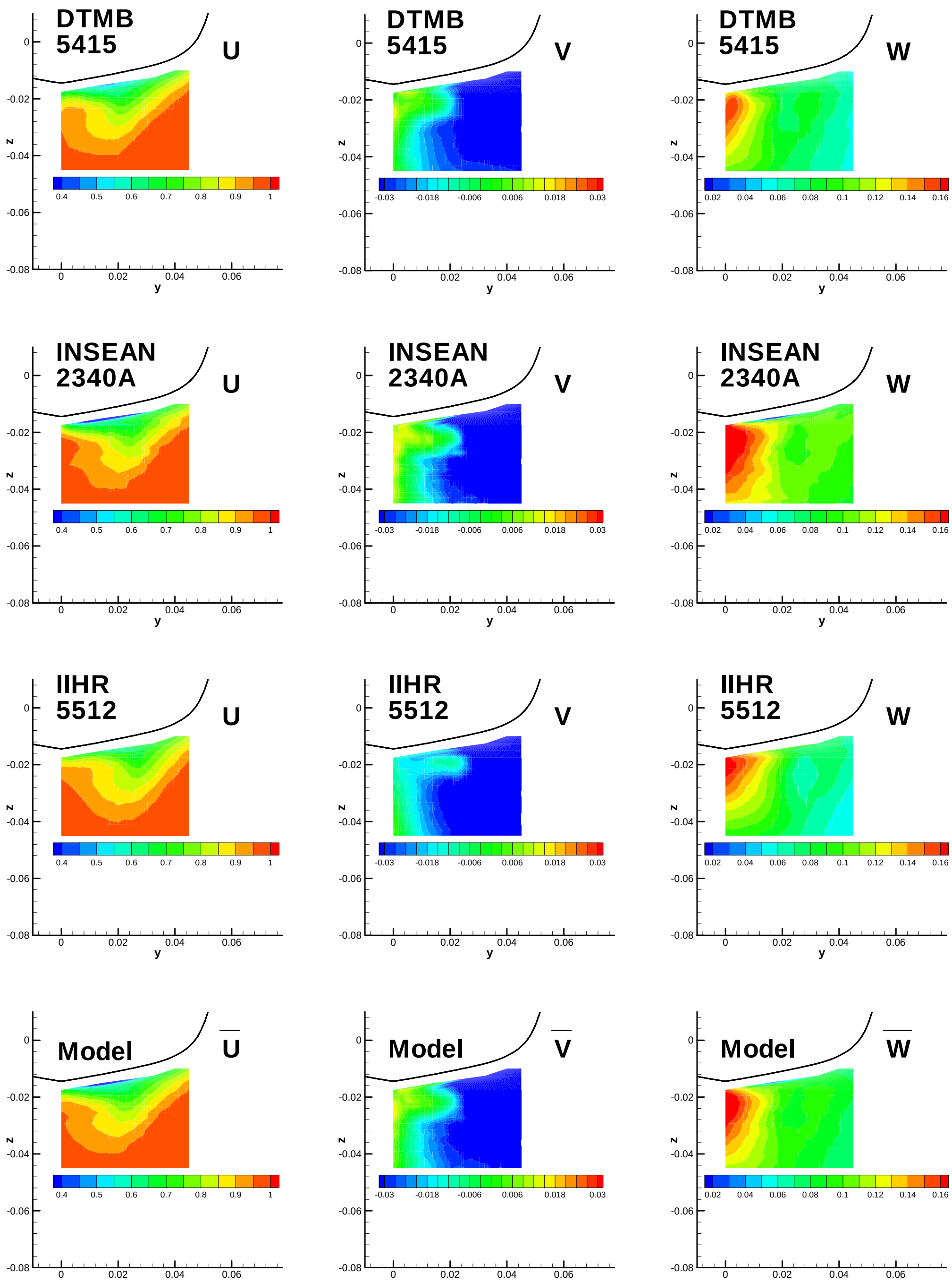


Figure B10 Nominal wake velocity results and the mean velocity result (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Nominal_Wake\DTMB_dpINTnwake.dat

(2) INSEAN data file: \23rdONRdata\Nominal_Wake\SEAN_dpINTnwake.dat

(3) IIHR data file: \23rdONRdata\Nominal_Wake\IIHR_dpINTnwake.dat

(4) Mean data file: \23rdONRdata\Nominal_Wake\MeanABC_UVW.dat

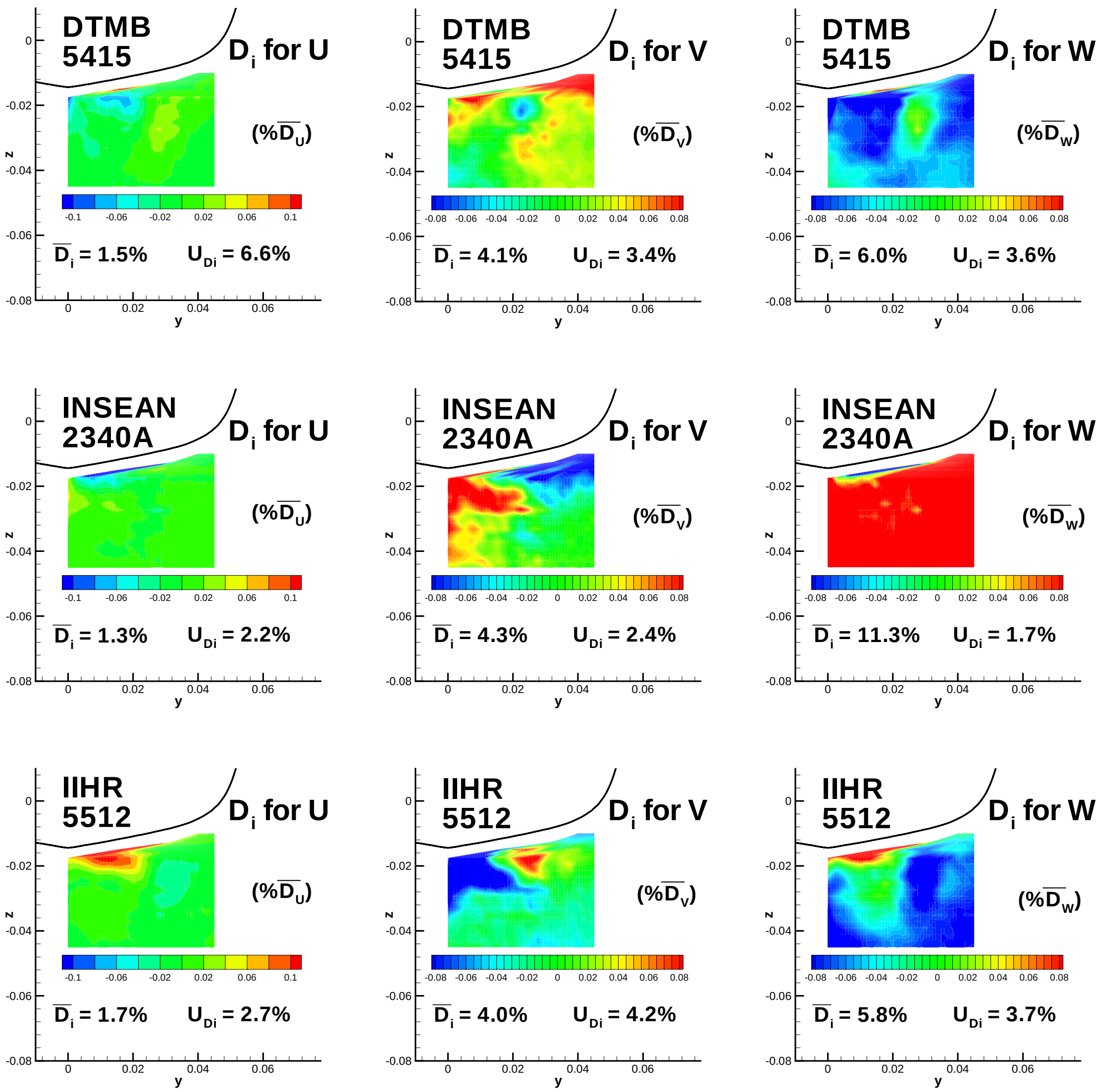


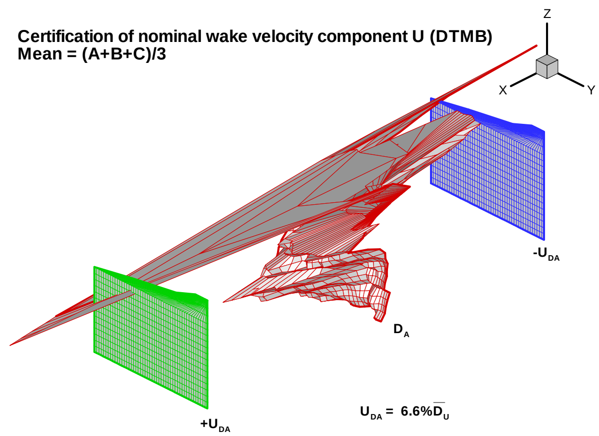
Figure B11 Contours of difference Di of nominal wake velocity results (Fr=0.28)

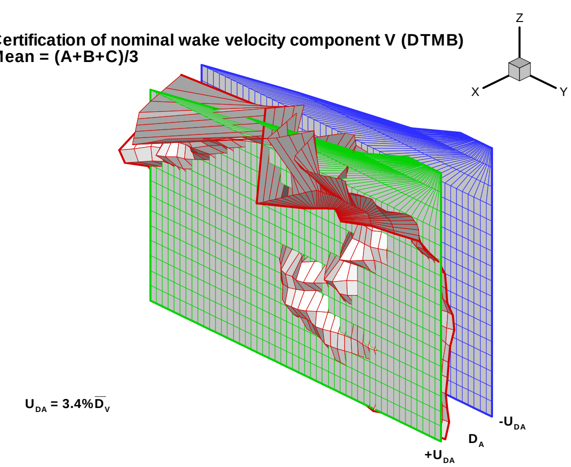
Note:

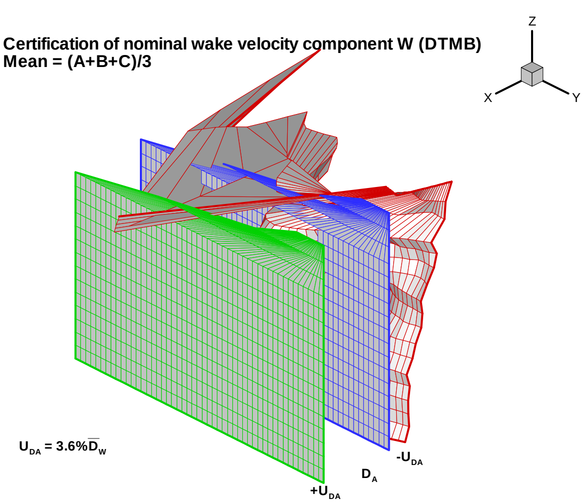
(1) DTMB data file: \23rdONRdata\Nominal_Wake\MeanABC_DA_perc.dat

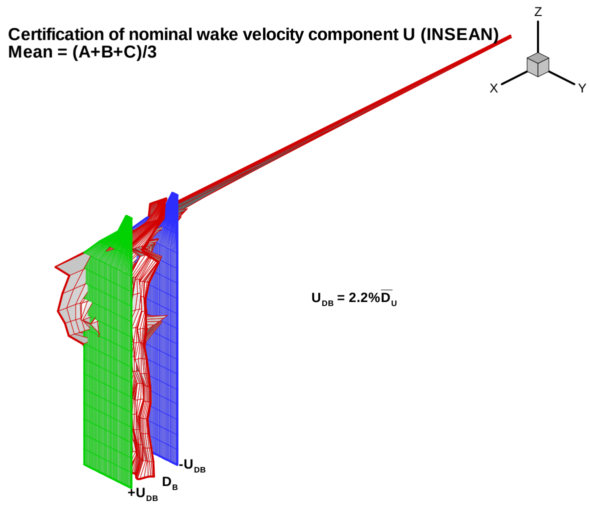
(2) INSEAN data file: \23rdONRdata\Nominal_Wake\MeanABC_DB_perc.dat

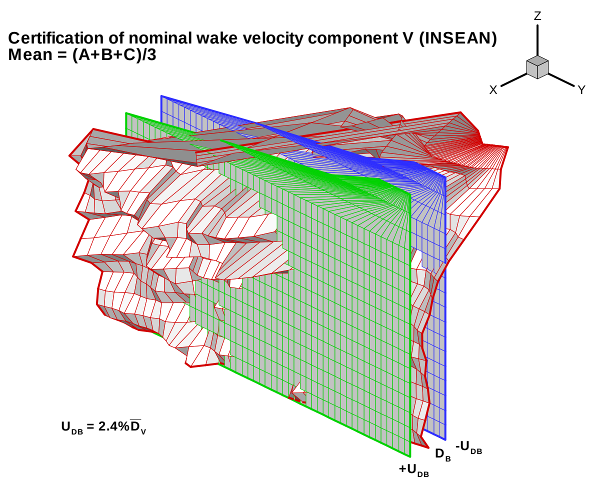
(3) IIHR data file: \23rdONRdata\Nominal_Wake\MeanABC_DC_perc.dat

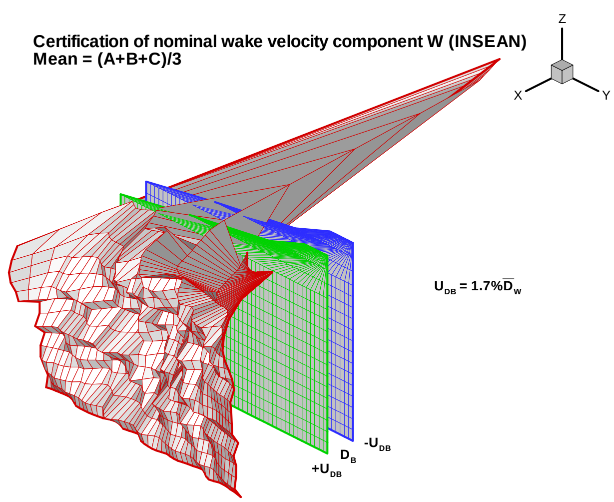

Certification of nominal wake velocity component U (DTMB)
Mean = (A+B+C)/3
Z
X
Y
$-U_{DA}$
D_A
$+U_{DA}$
$U_{DA} = 6.6\%\overline{D}_U$

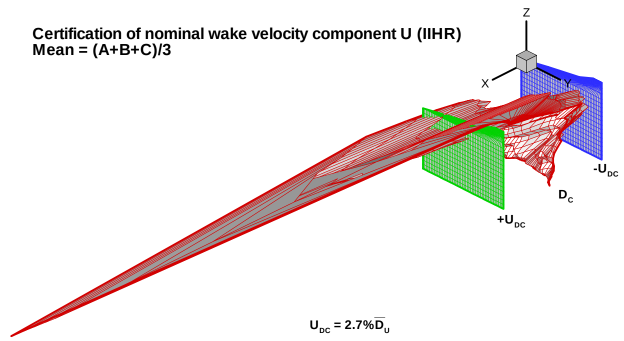

Certification of nominal wake velocity component V (DTMB)
Mean = (A+B+C)/3
Z
X
Y
$U_{DA} = 3.4\%\overline{D}_V$
$-U_{DA}$
D_A
$+U_{DA}$

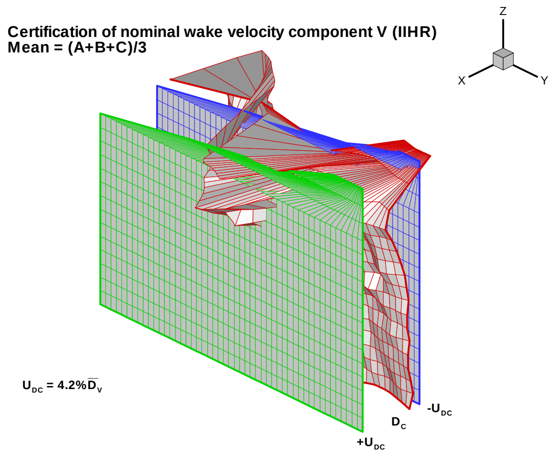

Certification of nominal wake velocity component W (DTMB)
Mean = (A+B+C)/3
Z
X
Y
$U_{DA} = 3.6\%\overline{D}_W$
$-U_{DA}$
D_A
$+U_{DA}$


Certification of nominal wake velocity component U (INSEAN)
Mean = (A+B+C)/3
Z
X
Y
$U_{DB} = 2.2\%\overline{D}_U$
$-U_{DB}$
D_B
$+U_{DB}$


Certification of nominal wake velocity component V (INSEAN)
Mean = (A+B+C)/3
Z
X
Y
$U_{DB} = 2.4\%\overline{D}_V$
$-U_{DB}$
D_B
$+U_{DB}$


Certification of nominal wake velocity component W (INSEAN)
Mean = (A+B+C)/3
Z
X
Y
$U_{DB} = 1.7\%\overline{D}_W$
$-U_{DB}$
D_B
$+U_{DB}$





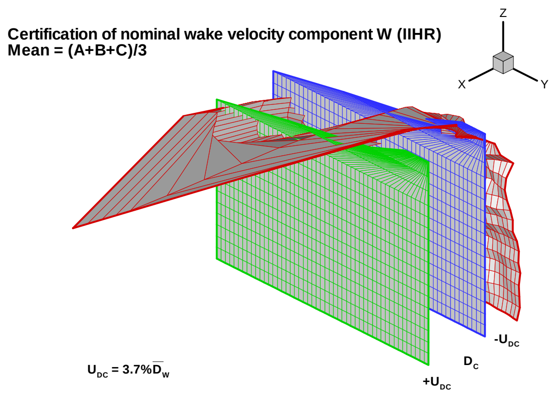


Figure B12 3D surface of difference Di of nominal wake velocity results (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Nominal_Wake\MeanABC_DA.dat

(2) INSEAN data file: \23rdONRdata\Nominal_Wake\MeanABC_DB.dat

(3) IIHR data file: \23rdONRdata\Nominal_Wake\MeanABC_DC.dat

APPENDIX C: FACILITY CERTIFICATION/BIASES FOR SINKAGE, TRIM, WAVE PROFILE, WAVE ELEVATION, AND NOMINAL WAKE VELOCITY USING AVERAGE $\overline{X}$ = (A+B)/2

Note for Table 3 in text:

(1) X_i for A and C from Interpolated data files (dtmb_intp.tec, iihr_intp.tec)

(2) X_i for B from IIHR Report 421 for INSEAN

(3) All $B^2_{X_i}$ and $P^2_{X_i}$ from [16]

(4) U_{X_i} for A from DTMB's Raw data files (dtmb_resist.tec)

(5) U_{X_i} for B from Angelo Olivieri's email to Fred Stern (04/08/2003)

(6) U_{X_i} for C from IIHR Raw data document

Table C1. Facility certification or biases for sinkage results σ

Fr & Facility		N-order level (% X_i)				M×N-order level (% $\overline{X}$)				Facility Certification or biases (% $\overline{X}$)			
		X_i	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\overline{X}}$	$P^2_{\overline{X}}$	$U_{\overline{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	7.49E-04	75.6%	24.4%	12.2%	12.6%	6.7%	93.3%	21.2%	3.5%	24.7%	0	12.6%
	B	6.98E-04	0.0%	100.0%	42.0%	40.5%				-3.5%	45.7%	0	40.5%
	C	5.93E-04	82.2%	17.8%	8.7%	7.1%				-18.0%	22.4%	0	7.1%
	AVE	7.23E-04	52.6%	47.4%	21.0%	20.1%				-6.0%	30.9%	0	20.1%
0.28	A	7.35E-03	68.4%	32.6%	5.6%	5.6%	39.6%	60.4%	3.7%	-0.3%	6.7%	0	5.6%
	B	7.39E-03	0.0%	100.0%	4.7%	4.7%				0.3%	6.0%	0	4.7%
	C	7.51E-03	30.4%	69.6%	1.4%	1.4%				1.9%	3.9%	0	1.4%
	AVE	7.37E-03	32.9%	67.4%	3.9%	3.9%				0.6%	5.5%	0	3.9%
0.41	A	1.73E-02	56.3%	44.7%	2.5%	2.4%	21.4%	78.6%	1.9%	-4.2%	3.1%	2.9%	3.7%
	B	1.88E-02	0.0%	100.0%	2.9%	3.1%				4.2%	3.6%	2.2%	3.7%
	C	1.85E-02	42.8%	57.2%	0.6%	0.6%				2.7%	2.0%	1.8%	1.9%
	AVE	1.80E-02	33.0%	67.3%	2.0%	2.0%				0.9%	2.9%	2.3%	3.1%

Note:

(1) DRE for sinkage in ONR paper figures for three institutes is

$$\sigma = \frac{2(\Delta FP + \Delta AP)}{L} \text{ instead of } \sigma = \frac{2}{Fr^2}\frac{2(\Delta FP + \Delta AP)}{2L}$$

(2) Uncertainties in terms of percentages remain the same for both above DREs, while uncertainty magnitudes are different as long as we assume that Fr number has no uncertainty for simplicity.

(3) X_i **for A, B, and C from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)**

(4) All $B^2_{X_i}$ **and** $P^2_{X_i}$ **from [16]**

(5) $B^2_{X_i}$ **and** $P^2_{X_i}$ **for DTMB (Fr=0.28 and 0.41) are best estimates from [16]**

(6) **All** U_{X_i} **based on mean values from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)**

Table C2. Facility certification or biases for trim results τ

Fr & Facility		N-order level (% X_i)				M×N-order level (% $\bar{X}$)				Facility Certification or biases (% $\bar{X}$)			
		X_i	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.10	A	-2.05E-04	64.5%	35.5%	14.4%	7.2%				-49.9%	25.3%	43.0%	43.6%
	B	-6.13E-04	0.0%	100.0%	32.0%	48.0%	1.4%	98.6%	24.3%	49.9%	53.8%	0	48.0%
	C	-6.22E-04	50.8%	49.2%	10.2%	15.6%				52.3%	28.8%	43.7%	46.4%
	AVE	-4.09E-04	38.4%	61.6%	18.9%	23.6%				17.4%	36.0%	28.9%	46.0%
0.28	A	-3.90E-03	54.7%	46.3%	2.8%	2.8%				1.7%	3.9%	0	2.8%
	B	-3.77E-03	0.0%	100.0%	4.7%	4.6%	15.0%	85.0%	2.7%	-1.7%	5.4%	0	4.6%
	C	-4.75E-03	36.1%	63.9%	1.8%	2.3%				23.9%	3.5%	23.6%	23.7%
	AVE	-3.83E-03	30.3%	70.1%	3.1%	3.2%				8.0%	4.3%	7.9%	10.4%
0.41	A	1.36E-02	38.1%	61.9%	1.5%	1.5%				-1.6%	1.7%	0	1.5%
	B	1.40E-02	0.0%	100.0%	0.9%	0.9%	28.0%	72.0%	0.9%	1.6%	1.2%	1.0%	1.3%
	C	1.06E-02	4.1%	95.9%	1.8%	1.4%				-23.1%	1.6%	23.0%	23.0%
	AVE	1.38E-02	14.1%	85.9%	1.4%	1.2%				-7.7%	1.5%	8.0%	8.6%

Note:

(1) DRE for trim in ONR paper figures for three institutes is

$$\tau = \frac{2(\Delta AP - \Delta FP)}{L} \text{ instead of } \tau = \frac{2}{Fr^2}\frac{(\Delta AP - \Delta FP)}{L}$$

(2) Uncertainties in terms of percentages remain the same for both above DREs, while uncertainty magnitudes are different as long as we assume that Fr number has no uncertainty for simplicity.

(3) X_i for A, B, and C from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)

(4) All $B^2_{X_i}$ and $P^2_{X_i}$ from [16]

(5) $B^2_{X_i}$ and $P^2_{X_i}$ for DTMB (Fr=0.28) are best estimates from [16]

(6) **All U_{X_i} based on mean values from Interpolated data files (dtmb_intp.tec, sean_intp.tec, iihr_intp.tec)**

Table C3. Facility certification or biases for wave profile results ζ

Fr &	**Facility**	**N-order level (% D_{X_i})**					**M×N-order level (% $\overline{D_{X_i}}$)**				**Facility Certification or biases (% $\overline{D_{X_i}}$)**			
		X_i	D_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\overline{X}}$	$P^2_{\overline{X}}$	$U_{\overline{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	1.82E-02	1.89E-02	64.5%	35.5%	3.5%	3.2%				-4.4%	4.2%	1.4%	3.5%
	B	2.00E-02	2.09E-02	100.0%	0.0%	4.2%	4.2%	87.0%	13.0%	2.7%	4.4%	5.0%	0	4.2%
	C	2.03E-02	2.22E-02	83.7%	16.3%	3.4%	3.7%				5.9%	4.5%	3.8%	5.3%
	AVE	1.91E-02	2.07E-02	82.7%	17.3%	3.7%	3.7%				2.0%	4.6%	1.7%	4.3%
0.41	A	3.02E-02	3.62E-02	64.5%	35.5%	1.8%	2.0%				3.1%	2.4%	1.9%	2.8%
	B	2.82E-02	2.46E-02	100.0%	0.0%	2.6%	1.9%	81.5%	18.5%	1.4%	-3.1%	2.4%	2.0%	2.8%
	C	3.03E-02	3.86E-02	81.6%	18.4%	2.0%	2.3%				3.3%	2.7%	1.9%	3.0%
	AVE	2.92E-02	3.32E-02	82.0%	18.0%	2.1%	2.1%				1.1%	2.5%	1.9%	2.9%

X_i **is the maximum elevation on the wave profile.**

Note:

(1) X_i **from Data files before interpolation (dtmb_dpwpro28.tec, sean_dpwpro28.tec, iihr_dpwpro28.tec)**

(2) D_{X_i} **for A from DTMB's website: http://www50.dt.navy.mil/5415/profile.html, (0.15 inch for all points)**

(3) D_{X_i} **for B from IIHR Report 421 (p.53) for INSEAN**

(4) D_{X_i} **for C from IIHR's raw data files (wpro2801.tec, wpro2802.tec, wpro2803.tec, wpro4101.tec, wpro4102.tec, wpro4103.tec)**

(5) All $B^2_{X_i}$ **and** $P^2_{X_i}$ **from [16]**

(6) All U_{X_i} **from [16]**

Table C4. Facility certification or biases for wave elevation results ζ at cut y=0.324

Fr & Facility		N-order level (% D_{X_i})					M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
		X_i	D_{X_i}	$B_{X_i}^2$	$P_{X_i}^2$	U_{X_i}	U_{X_i}	$B_{\bar{X}}^2$	$P_{\bar{X}}^2$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	4.19E-03	1.17E-02	76.6%	23.4%	2.7%	2.8%	71.9%	28.1%	1.8%	-4.5%	3.4%	3.0%	4.1%
	B	5.20E-03	1.09E-02	64.9%	35.1%	2.4%	2.3%				4.5%	3.0%	3.4%	4.1%
	C	3.79E-03	1.11E-02	59.0%	41.0%	3.4%	3.4%				-8.1%	3.8%	7.1%	7.9%
	AVE	4.70E-03	1.12E-02	66.8%	33.2%	2.9%	2.9%				-2.7%	3.4%	4.5%	5.4%

X_i **is the maximum elevation on the cut y=0.324.**

Note:

(1) X_i **from Interpolated data files (dtmb_int324.tec, insean_int324.tec, iihr_int324.tec)**

(2) D_{X_i} **for A from Interpolated data files (dtmb_intpat.tec)**

(3) D_{X_i} **for B from IIHR Report 421 (p.54) for INSEAN**

(4) D_{X_i} **for C from (Joe Longo) IIHR's raw data files (a0_101.dat zone = "Steady")**

(5) All $B_{X_i}^2$ **and** $P_{X_i}^2$ **from [16]**

(6) All U_{X_i} **from [16]**

(7) $B_{X_i}^2$ **and** $P_{X_i}^2$ **for DTMB (Fr=0.28) are best estimates from [16]**

Table C5. Facility certification or biases for wave elevation results ζ

Fr & Facility		N-order level (% D_{X_i})				M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
		D_{X_i}	$B_{X_i}^2$	$P_{X_i}^2$	U_{X_i}	U_{X_i}	$B_{\bar{X}}^2$	$P_{\bar{X}}^2$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
0.28	A	1.17E-02	76.6%	23.4%	2.7%	2.8%				3.4%	3.4%	0	2.8%
	B	1.09E-02	64.9%	35.1%	2.4%	2.3%	71.9%	28.1%	1.8%	3.4%	3.0%	1.6%	2.8%
	C	1.11E-02	59.0%	41.0%	3.4%	3.4%				2.9%	3.8%	0	3.4%
	AVE	1.12E-02	66.8%	33.2%	2.9%	2.9%				3.2%	3.4%	0.6%	3.0%

Note:

(1) D_{X_i} **for A from Interpolated data files (dtmb_intpat.tec)**

(2) D_{X_i} **for B from IIHR Report 421 (p.54) for INSEAN**

(3) D_{X_i} **for C from (Joe Longo) IIHR's raw data files (a0_101.dat zone = "Steady")**

(4) All $B_{X_i}^2$ **and** $P_{X_i}^2$ **from [16]**

(5) All U_{X_i} **from [16]**

(6) $B_{X_i}^2$ **and** $P_{X_i}^2$ **for DTMB (Fr=0.28) are best estimates from [16]**

(7)All D_i **calculated from Interpolated data files (dtmb_intpat.tec, insean_intpat.tec, iihr_intpat.tec)**

Table C6. Facility certification or biases for nominal wake velocity results at Fr=0.28

Fr & Facility		N-order level (% D_{X_i})											M×N-order level (% $\overline{D_{X_i}}$)				Facility Certification or biases (% $\overline{D_{X_i}}$)			
			LTR			HTR			SQRT (LTR^2+HTR^2)/2										
		D_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	$B^2_{X_i}$	$P^2_{X_i}$	U_{X_i}	U_{X_i}	$B^2_{\bar{X}}$	$P^2_{\bar{X}}$	$U_{\bar{X}}$	D_i	U_{D_i}	U_{FB_i}	U_{T_i}
	A	1.00E+00	65.4%	34.4%	1.6%	74.5%	26.5%	12.5%	73.6%	26.4%	6.3%	6.3%				1.0%	7.0%	0	6.3%
0.28 U	B	1.02E+00	47.8%	52.2%	0.4%	60.4%	39.6%	0.4%	53.5%	46.5%	0.3%	0.3%	73.6%	26.4%	3.1%	1.0%	3.2%	0	0.3%
	C	1.01E+00	99.8%	0.2%	1.2%	99.2%	0.8%	3.1%	99.3%	0.7%	1.7%	1.7%				2.5%	3.6%	0	1.7%
	AVE	1.01E+00	71.0%	28.9%	1.1%	78.0%	22.3%	5.3%	75.5%	24.5%	2.8%	2.7%				1.5%	4.6%	0	2.7%
	A	9.62E-02	54.3%	46.7%	2.9%	43.5%	56.5%	6.5%	45.2%	54.8%	3.6%	3.0%				3.6%	3.4%	1.2%	3.2%
0.28 V	B	1.19E-01	21.2%	78.8%	1.9%	15.9%	84.1%	2.7%	17.6%	82.4%	1.6%	1.7%	38.4%	61.6%	1.7%	3.6%	2.4%	2.7%	3.2%
	C	1.33E-01	99.7%	0.3%	5.5%	93.3%	6.7%	3.8%	97.7%	2.3%	3.3%	3.8%				6.0%	4.2%	4.3%	5.8%
	AVE	1.16E-01	58.4%	41.9%	3.4%	50.9%	49.1%	4.3%	53.5%	46.5%	2.9%	2.8%				4.4%	3.3%	2.7%	4.0%
	A	1.47E-01	65.3%	34.7%	6.5%	44.6%	35.4%	3.7%	63.3%	36.7%	3.6%	3.3%				8.6%	3.7%	7.8%	8.5%
0.28 W	B	1.65E-01	79.1%	20.9%	1.0%	87.9%	12.1%	0.9%	83.1%	16.9%	0.6%	0.6%	64.1%	35.9%	1.7%	8.6%	1.8%	8.4%	8.5%
	C	1.79E-01	99.9%	0.1%	4.1%	99.2%	0.8%	4.5%	99.5%	0.5%	3.0%	3.3%				8.8%	3.7%	7.9%	8.6%
	AVE	1.64E-01	81.4%	18.6%	3.8%	77.2%	16.1%	3.0%	82.0%	18.0%	2.4%	2.4%				8.7%	3.1%	8.1%	8.5%

Note:

(1) D_{X_i} **for A from DTMB Raw data files: (dtmb_dpnwake.tec)**

(2) D_{X_i} **for B from IIHR Report 421 (p.56) for INSEAN, but dynamic range for U is divided by 2.**

(3) D_{X_i} **for C from IIHR Raw data document**

(4) All $B_{X_i}^2$ and $P_{X_i}^2$ from [16]

(5) All U_{X_i} from [16]

(6) $B_{X_i}^2$ and $P_{X_i}^2$ for DTMB are best estimates from [16]

(7) All D_i calculated from Interpolated data files (dtmb_dpINTnwake.tec, sean_dpINTnwake.tec, iihr_dpINTnwake.tec)

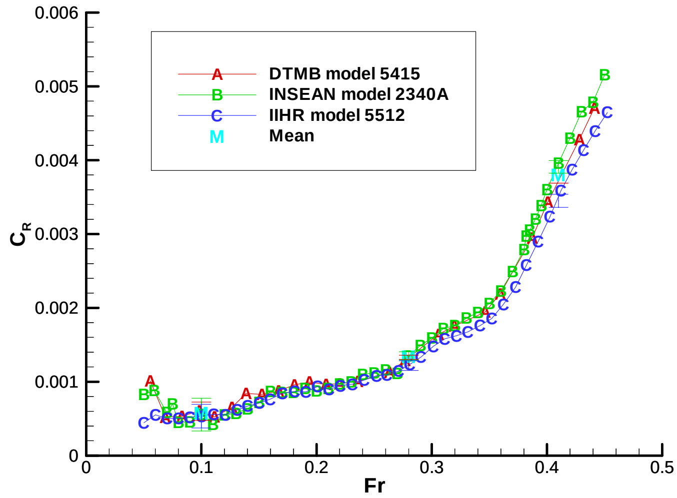


Figure C1 Residuary resistance results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Resistance\dtmb_dpcr.tec

(2) INSEAN data file: \23rdONRdata\Resistance\sean_dpcr.tec

(3) IIHR data file: \23rdONRdata\Resistance\iihr_dpcr.tec

(4) Mean data file: \23rdONRdata\Resistance\meanAB.txt

(5) DTMB uncertainty file: \23rdONRdata\Resistance\dtmb_dpcr_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Resistance\sean_dpcr_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Resistance\iihr_dpcr_un.dat

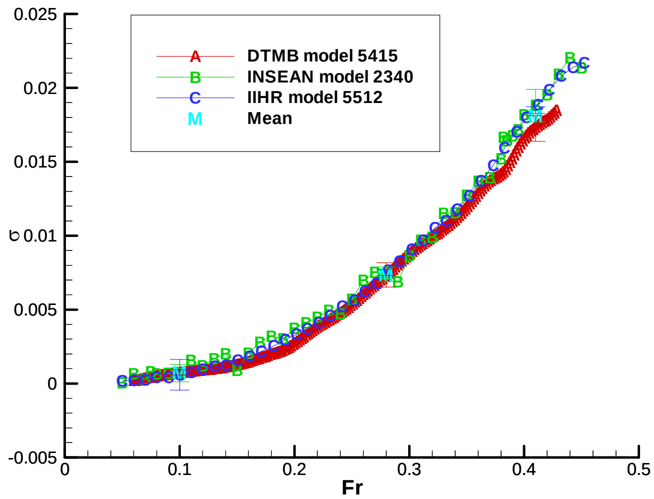


Figure C2 Sinkage results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Sinkage_Trim\dtmb_dpst.tec

(2) INSEAN data file: \23rdONRdata\Sinkage_Trim\sean_dpst.tec

(3) IIHR data file: \23rdONRdata\Sinkage_Trim\iihr_dpst.tec

(4) Mean data file: \23rdONRdata\Sinkage_Trim\meanAB.txt

(5) DTMB uncertainty file: \23rdONRdata\Sinkage_Trim\dtmb_dpcr_sink_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Sinkage_Trim\sean_dpcr_sink_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Sinkage_Trim\iihr_dpcr_sink_un.dat

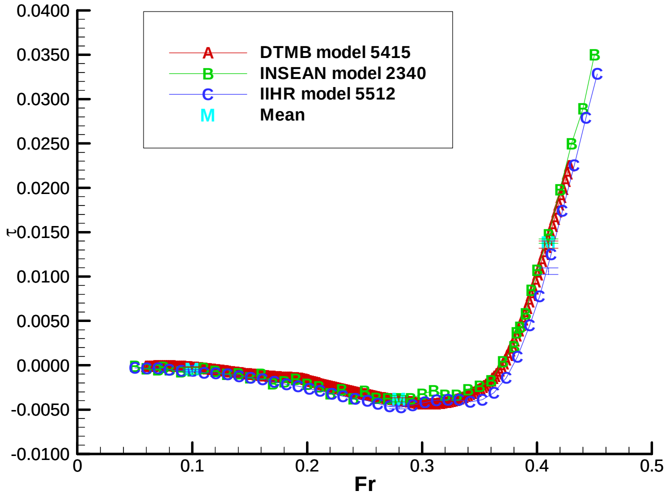


Figure C3 Trim results and uncertainty bands at Fr=0.1, 0.28 and 0.41

Note:

(1) DTMB data file: \23rdONRdata\Sinkage_Trim\dtmb_dpst.tec

(2) INSEAN data file: \23rdONRdata\Sinkage_Trim\sean_dpst.tec

(3) IIHR data file: \23rdONRdata\Sinkage_Trim\iihr_dpst.tec

(4) Mean data file: \23rdONRdata\Sinkage_Trim\meanAB.txt

(5) DTMB uncertainty file: \23rdONRdata\Sinkage_Trim\dtmb_dpcr_trim_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Sinkage_Trim\sean_dpcr_trim_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Sinkage_Trim\iihr_dpcr_trim_un.dat

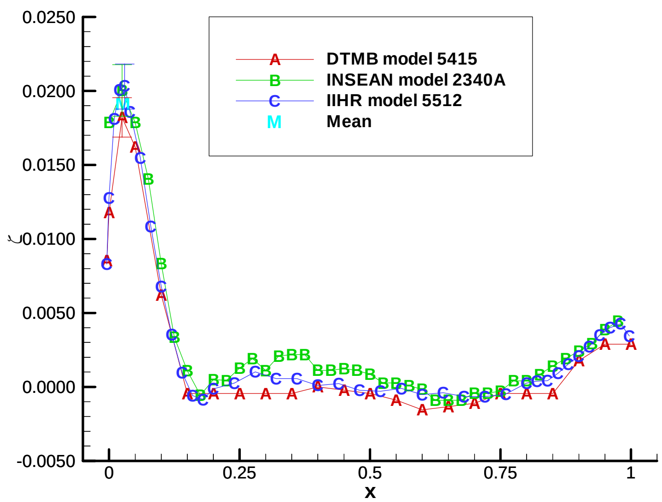


Figure C4 Wave profile results and uncertainty bands at ζ_{max} (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Profiles\dtmb_dpwpro28.tec

(2) INSEAN data file: \23rdONRdata\Wave_Profiles\sean_dpswpro28.tec

(3) IIHR data file: \23rdONRdata\Wave_Profiles\iihr_dpwpro28.tec

(4) Mean data file: \23rdONRdata\Wave_Profiles\mean28AB.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Profiles\dtmb_dp28_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Profiles\sean_dp28_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Profiles\iihr_dp28_un.dat

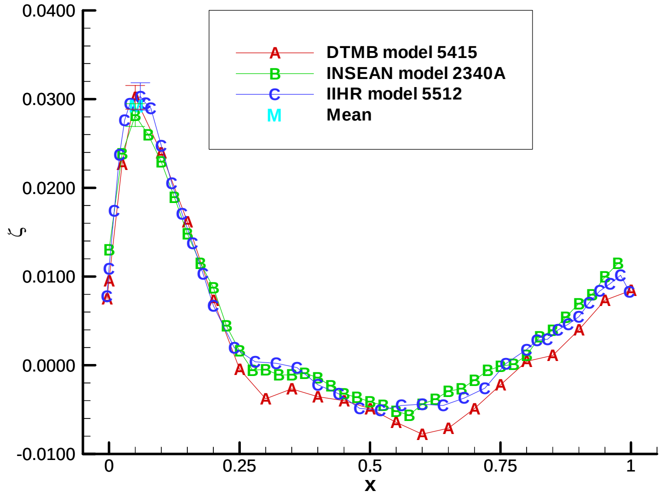


Figure C5 Wave profile results and uncertainty bands at $\zeta_{\max}$ (Fr=0.41)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Profiles\dtmb_dpwpro41.tec

(2) INSEAN data file: \23rdONRdata\Wave_Profiles\sean_dpswpro41.tec

(3) IIHR data file: \23rdONRdata\Wave_Profiles\iihr_dpwpro41.tec

(4) Mean data file: \23rdONRdata\Wave_Profiles\mean41AB.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Profiles\dtmb_dp41_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Profiles\sean_dp41_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Profiles\iihr_dp41_un.dat

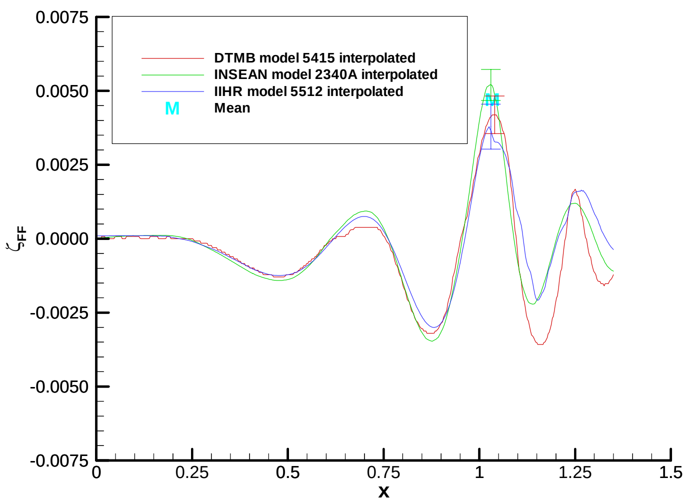


Figure C6 Wave elevation results and uncertainty bands for cut y=0.324 (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\dtmb_int324.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\sean_int324.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\iihr_int324.tec

(4) Mean data file: \23rdONRdata\Wave_Elevations\meanAB.txt

(5) DTMB uncertainty file: \23rdONRdata\Wave_Elevations\dtmb_dp28_un.dat

(6) INSEAN uncertainty file: \23rdONRdata\Wave_Elevations\sean_dp28_un.dat

(7) IIHR uncertainty file: \23rdONRdata\Wave_Elevations\iihr_dp28_un.dat

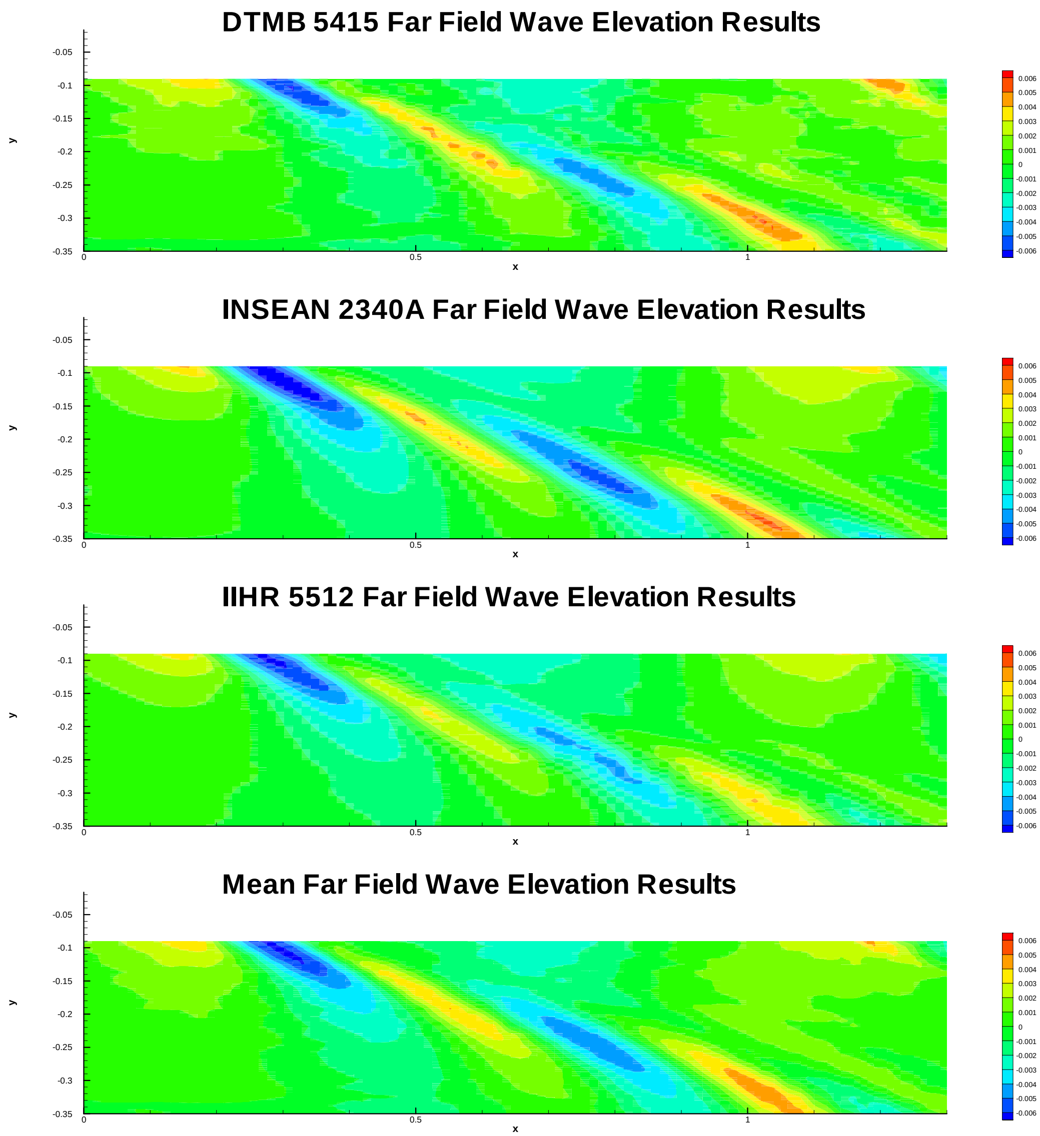


Figure C7 Wave elevation results for far field and the mean result (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\dtmb_intpat.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\insean_intpat.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\iihr_intpat.tec

(4) Mean data file: \23rdONRdata\Wave_Elevations\mean_Zeta_AB.dat

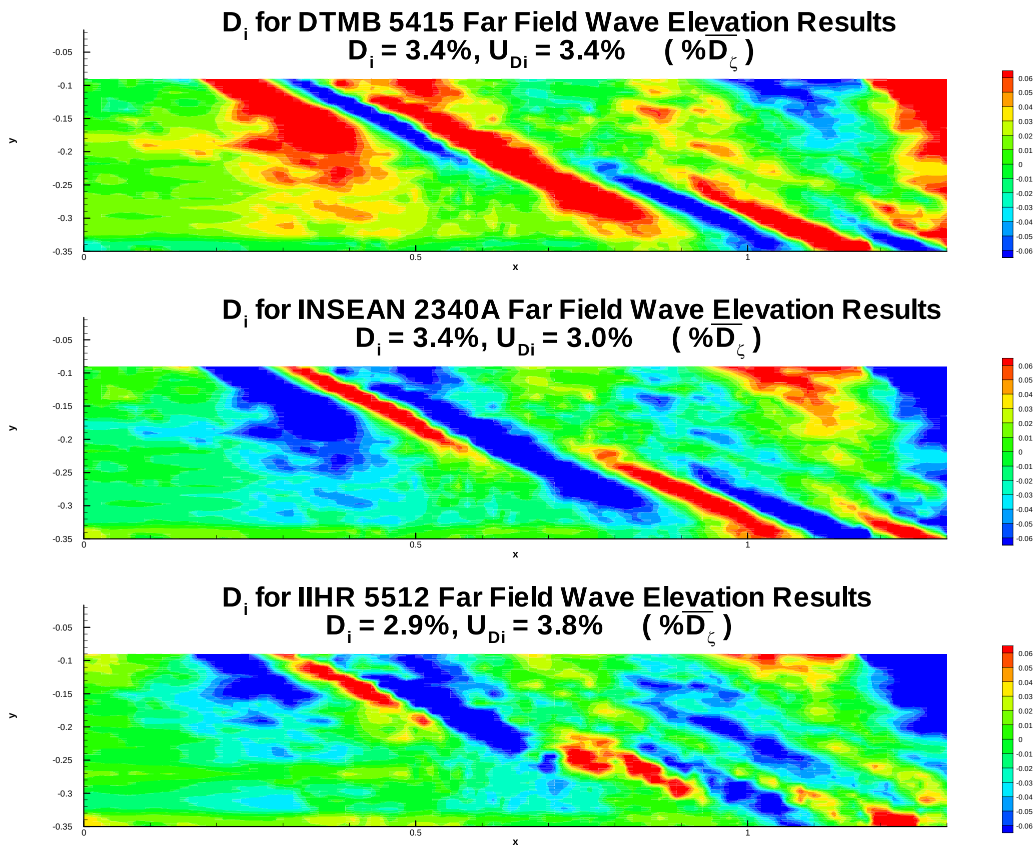


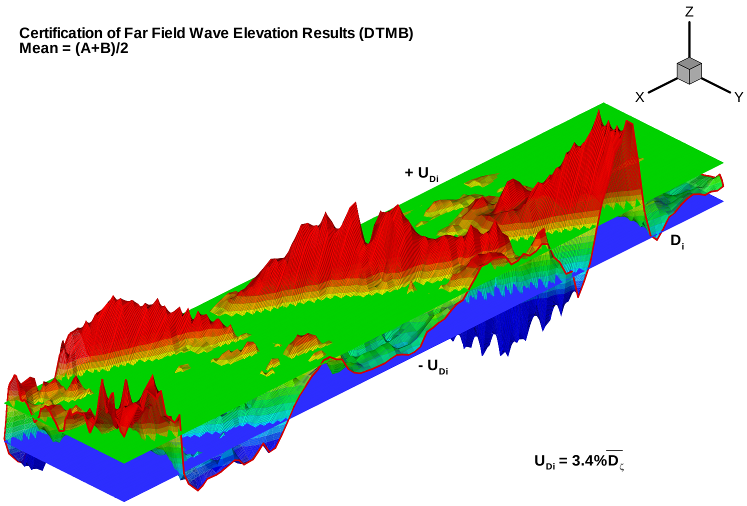
Figure C8 Contours of difference Di of wave elevation results (Fr=0.28)

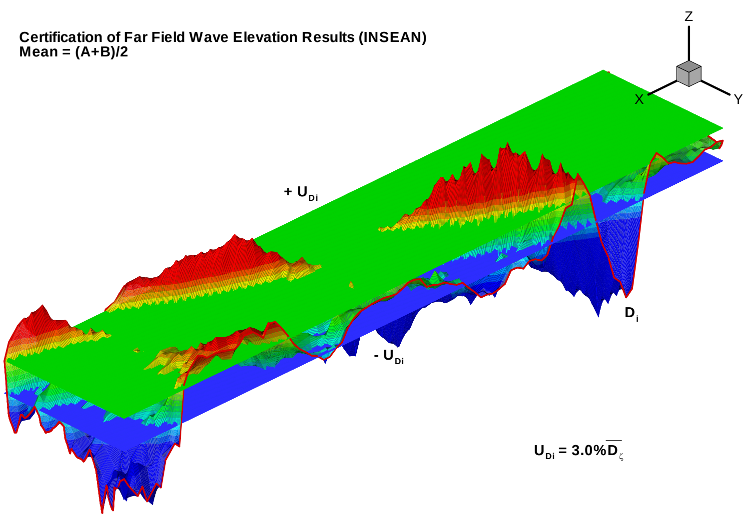
Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\DTMB_DZeta_AB.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\SEAN_DZeta_AB.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\IIHR_DZeta_AB.tec


Certification of Far Field Wave Elevation Results (DTMB)
Mean = (A+B)/2
Z
X
Y
+ U_{Di}
D_i
- U_{Di}
U_{Di} = 3.4%$\overline{D}_\zeta$


Certification of Far Field Wave Elevation Results (INSEAN)
Mean = (A+B)/2
Z
X
Y
+ U_{Di}
D_i
- U_{Di}
U_{Di} = 3.0%$\overline{D}_\zeta$

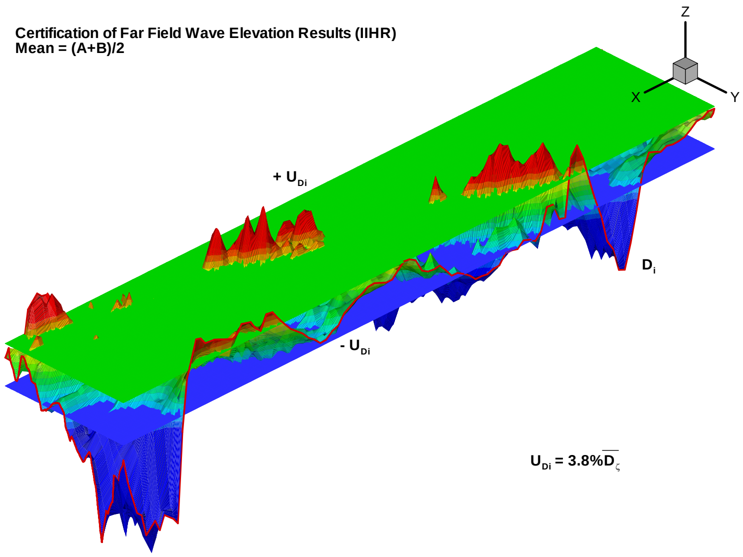


Figure C9 3D surface of difference Di of wave elevation results (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Wave_Elevations\DTMB_DZeta_AB.tec

(2) INSEAN data file: \23rdONRdata\Wave_Elevations\SEAN_DZeta_AB.tec

(3) IIHR data file: \23rdONRdata\Wave_Elevations\IIHR_DZeta_AB.tec

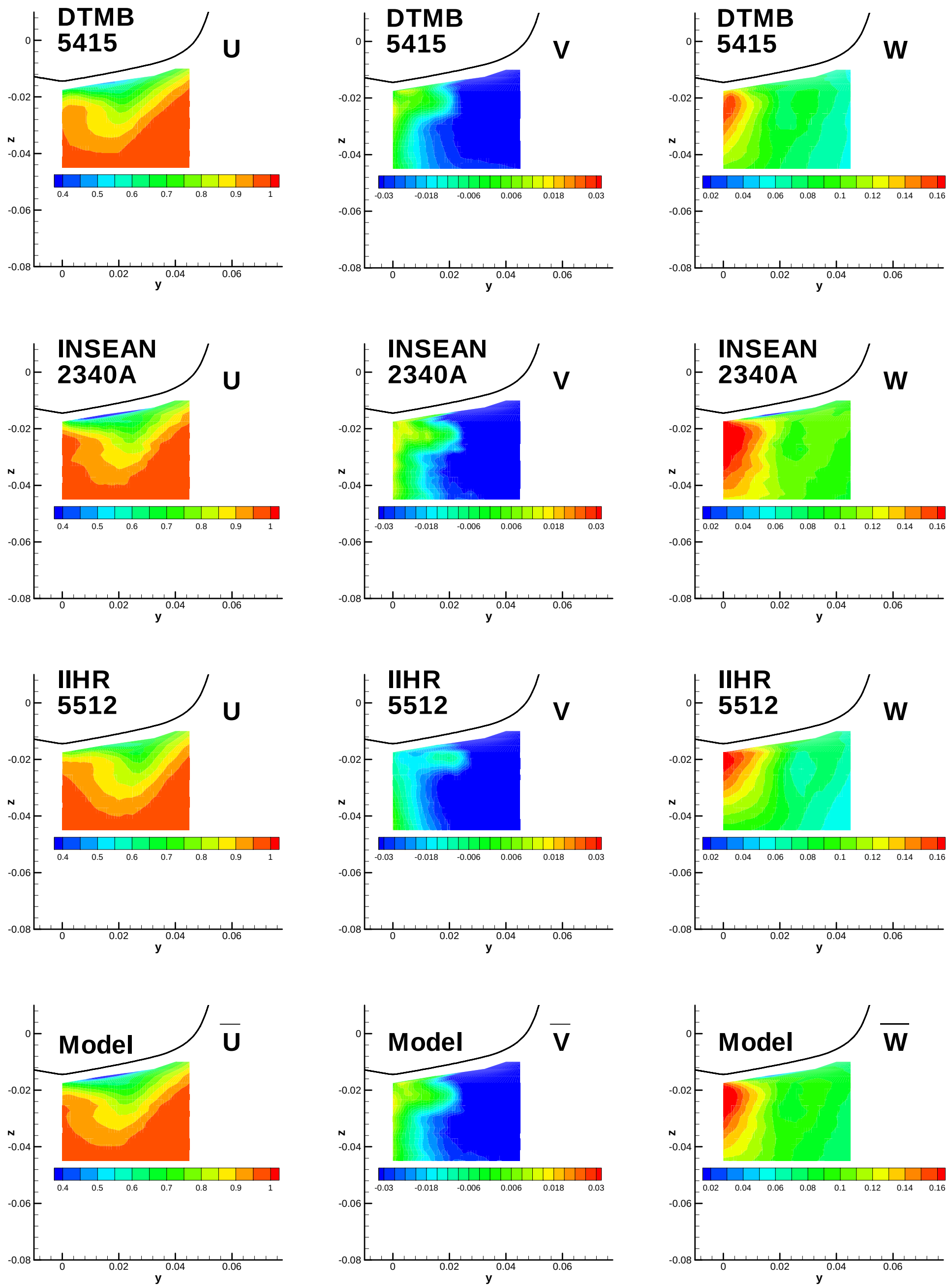


Figure C10 Nominal wake velocity results and the mean velocity result (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Nominal_Wake\DTMB_dpINTnwake.dat

(2) INSEAN data file: \23rdONRdata\Nominal_Wake\SEAN_dpINTnwake.dat

(3) IIHR data file: \23rdONRdata\Nominal_Wake\IIHR_dpINTnwake.dat

(4) Mean data file: \23rdONRdata\Nominal_Wake\MeanAB_UVW.dat

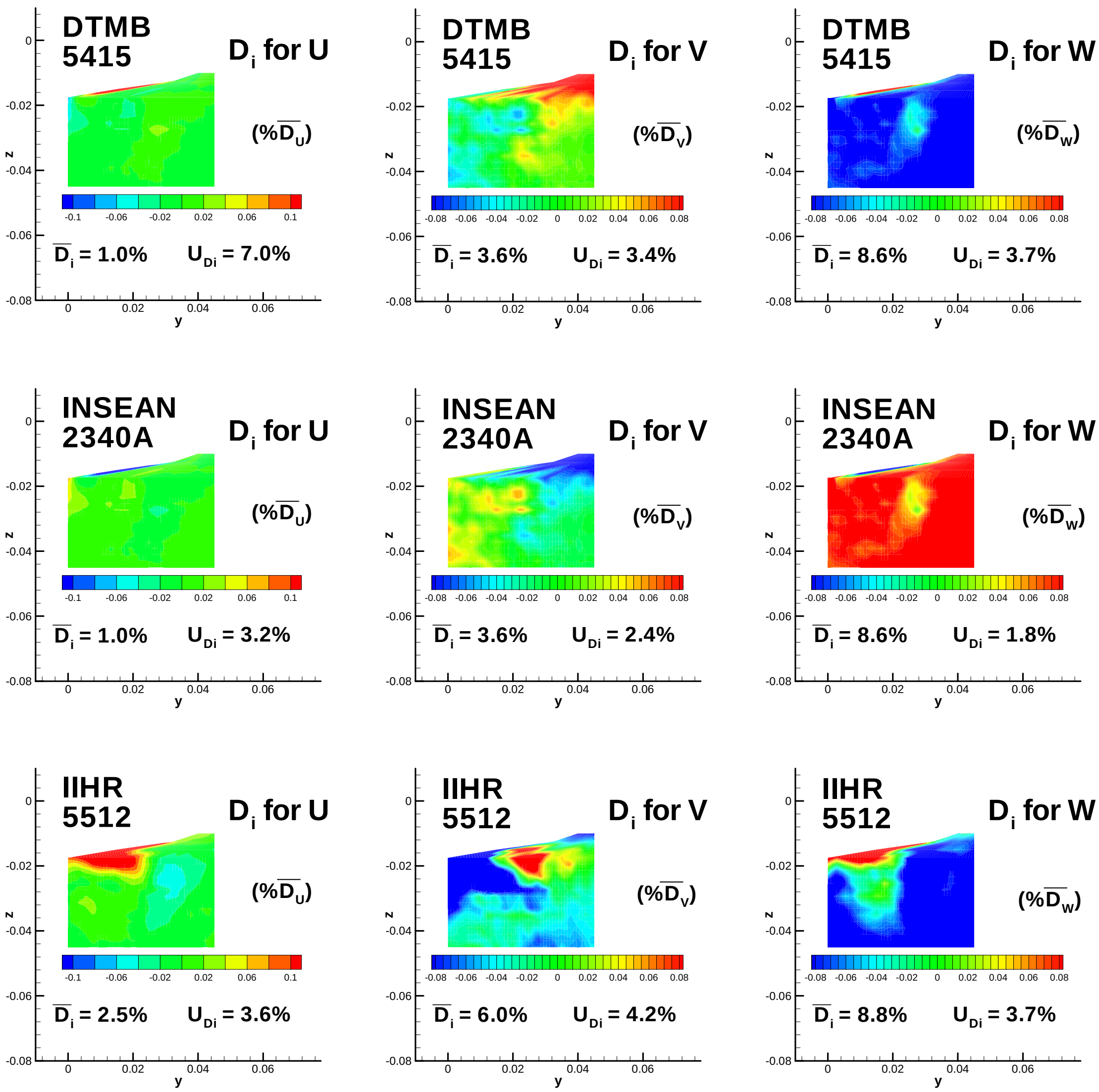


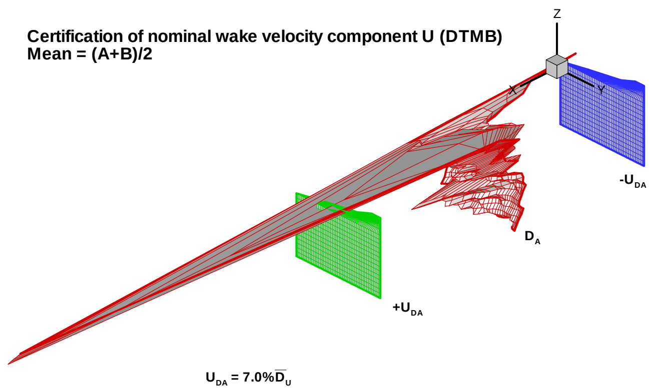
Figure C11 Contours of difference Di of nominal wake velocity results (Fr=0.28)

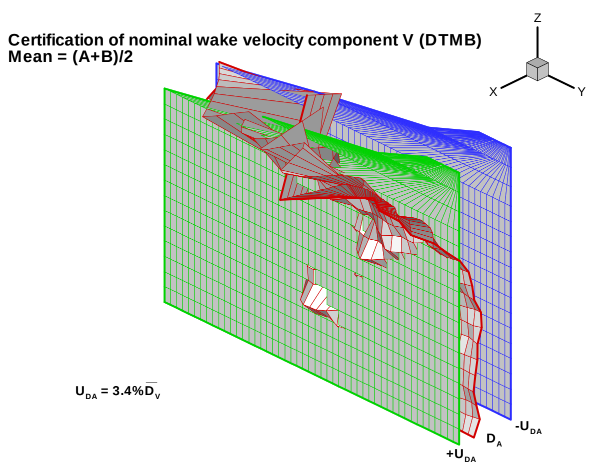
Note:

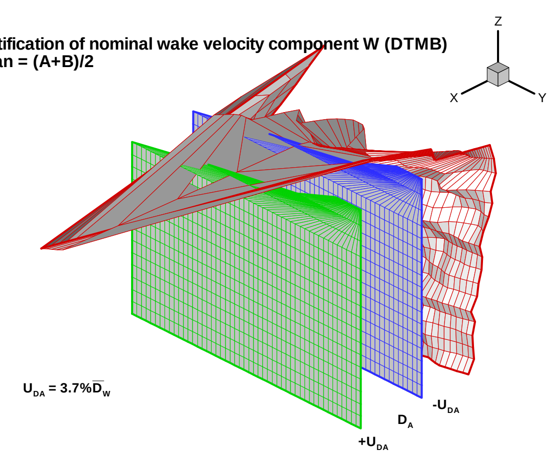
(1) DTMB data file: \23rdONRdata\Nominal_Wake\MeanAB_DA_perc.dat

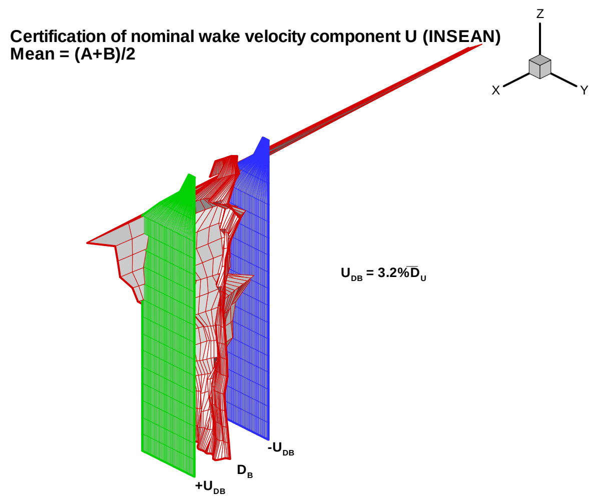
(2) INSEAN data file: \23rdONRdata\Nominal_Wake\MeanAB_DB_perc.dat

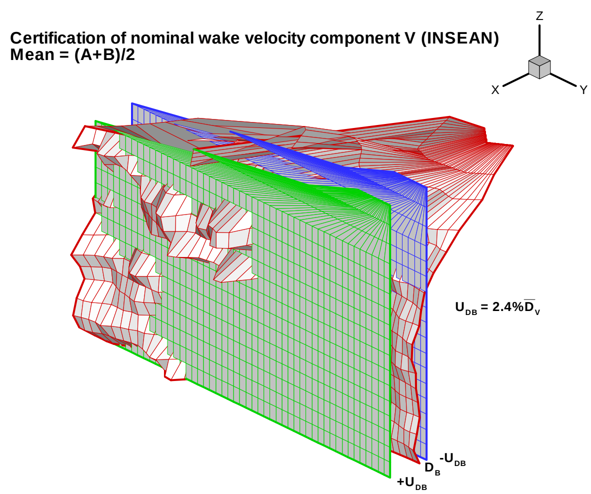
(3) IIHR data file: \23rdONRdata\Nominal_Wake\MeanAB_DC_perc.dat

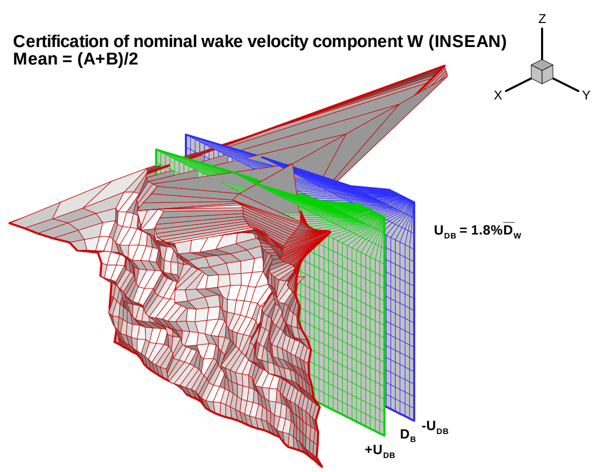

Certification of nominal wake velocity component U (DTMB)
Mean = (A+B)/2
Z
X
Y
-U_DA
D_A
+U_DA
U_DA = 7.0%D̄_U

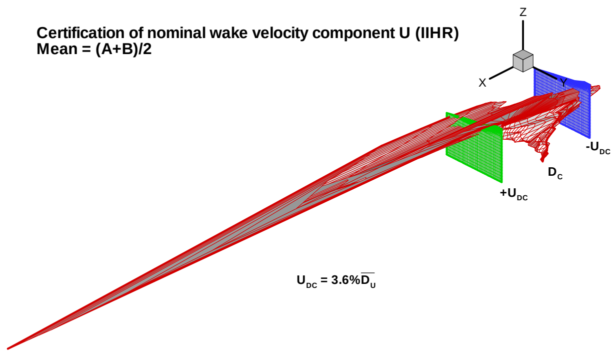

Certification of nominal wake velocity component V (DTMB)
Mean = (A+B)/2
Z
X
Y
U_DA = 3.4%D̄_V
-U_DA
D_A
+U_DA

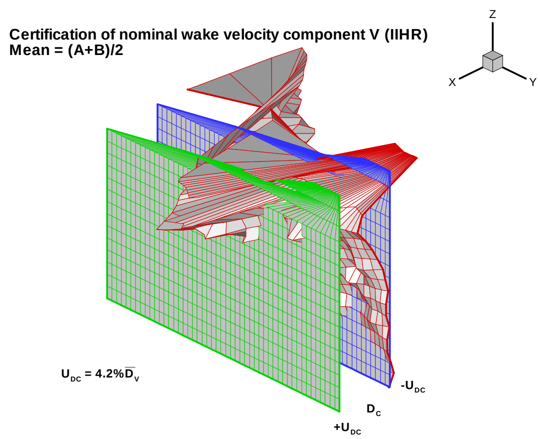

Certification of nominal wake velocity component W (DTMB)
Mean = (A+B)/2
Z
X
Y
U_DA = 3.7%D̄_W
-U_DA
D_A
+U_DA


Certification of nominal wake velocity component U (INSEAN)
Mean = (A+B)/2
Z
X
Y
$U_{DB} = 3.2\%\overline{D}_U$
$-U_{DB}$
D_B
$+U_{DB}$


Certification of nominal wake velocity component V (INSEAN)
Mean = (A+B)/2
Z
X
Y
$U_{DB} = 2.4\%\overline{D}_V$
$-U_{DB}$
D_B
$+U_{DB}$


Certification of nominal wake velocity component W (INSEAN)
Mean = (A+B)/2
Z
X
Y
$U_{DB} = 1.8\%\overline{D}_W$
$-U_{DB}$
D_B
$+U_{DB}$





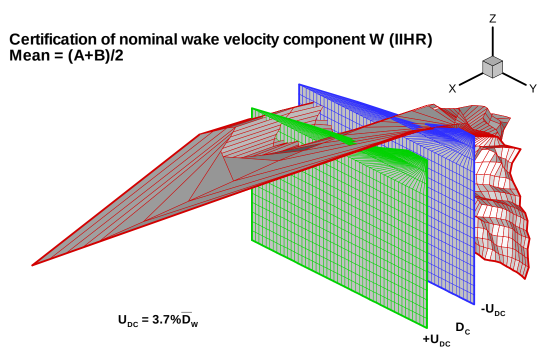


Figure C12 3D surface of difference Di of nominal wake velocity results (Fr=0.28)

Note:

(1) DTMB data file: \23rdONRdata\Nominal_Wake\MeanAB_DA.dat

(2) INSEAN data file: \23rdONRdata\Nominal_Wake\MeanAB_DB.dat

(3) IIHR data file: \23rdONRdata\Nominal_Wake\MeanABC_D.dat